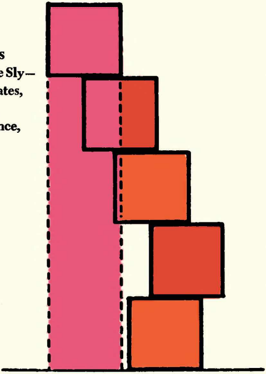
Mathematical Puzzles for the Connoisseur

P.M.H. Kendall
and
G.M. Thomas

104 Brainteasers
Some Funny, Some Sly— on Weights and Dates, Areas and Shapes, Numbers and Chance, Cards and Chess



Can you place one cube upon another, upon another, etc., until the plan view of one of them lies entirely outside the area of the base cube?

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Preface

Nearly all puzzle books published today grossly underestimate the intelligence of their public. The reader can no longer be entertained with the simple match-trick or coin puzzles, neither does he want to see problems purely mathematical in nature, such as can be found in any textbook. We have tried to produce a book of those puzzles which lie between the two extremes.

Although most of the problems can be solved by logic and common sense, some of them may also be solved more elegantly by mathematics.

The essential ingredients of a good puzzle are not hard to establish. It should be ingenious but clear, complete yet concise, if possible amusing, and have a unique solution. It is for our readers to decide how many of these puzzles bear comparison with this ideal.

Completely new ideas for puzzles are unfortunately rare and so we do not apologise for including a few of the "Aged but not infirm". Many of the problems are original and we claim to have improved others.

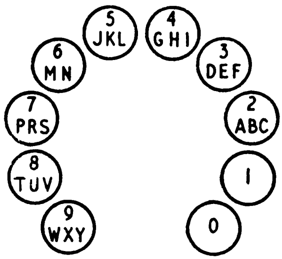
We take this opportunity to thank Dr. M. G. Kendall for allowing us to include some of his problems and for giving us constant advice. Thanks are also due to Mr. E. C. Lester and to the proprietors of the "Autocar" for some motoring puzzles, Mr. R. Martin for the triangular revolver duel puzzle, and to Anne and Barbara for their help in the preparation of this book.

P.M.H.K.
G.M.T.

Contents

General A

A/1



Pongo's telephone number is ANThill 4729, but I can never remember it. However, I don't have to bother if there is an automatic exchange because I can get it by dialling the word ANTHRAX, the letter H being in the same hole as the digit 4, and so on. This is a useful idea which might be extended. Can you find a word for our home number, TABernacle 2463, and office number, VINcent 8225?

A/2

Three men order a bottle of wine at 30*s*. to drink with their meal. They each pay 10*s*. for it. The waitress asks the manager, who only has a bottle at 25*s*., so he gives the waitress the wine and 5*s*. change. She returns to the table but gives only 3*s*. back, keeping 2*s*. for herself. Since each man paid 10*s*. and received 1*s*. change he has paid 9*s*. But $3 \times 9 = 27$*s*., which with the 2*s*. kept by the waitress makes 29*s*.

Where did the shilling go?

A/3

The word *anemone* is remarkable as having four syllables and only seven letters, equivalent to 1·75 letters only per syllable. Can you better this with a word of at least four syllables? No proper names are allowed. If *y* is pronounced as a vowel you count it as such; otherwise you count it as a consonant. We found two suitable words, with twice as many vowels as consonants. Which could they have been?

A/4

A road and a railway run parallel to each other until a bend brings the road to a level crossing. A man cycles to work along the road at a constant speed of 12 m.p.h. At the crossing he normally meets a train that travels in the same direction. One day he was 25 minutes late for work and met the train 6 miles before the crossing. What was the speed of the train?

A/5

Two cylinders, one of lead and the other of titanium, are identical in physical dimensions and are both painted green, so that you cannot tell which is which. They both weigh the same, the lead cylinder being hollow and the titanium solid. Of course, the hollow cylinder, being lead, does not sound hollow. How can you distinguish between the two without scratching or damaging either cylinder and without using any other object?

A/6

A barge floating in a canal lock is loaded with cubes of ice. A man on the barge unloads the ice into the water and of course it melts. Will the water level in the lock rise, fall, or remain steady?

Assuming all the ice at the North Pole to be floating in the sea, what would happen to 'sea level' should all the ice melt at the Pole?

A/7

Quite often in 'Western' pictures the wheels of stage-coaches appear to be rotating backwards. In a film we saw recently the wheels appeared to be stationary although the horses were galloping 'full out'. Fascinated by this, we counted the number of spokes and found there were twelve per wheel. The six-foot hero stood about twice as tall as the wheels, so we estimated that the wheel was three feet in diameter. If the film was being shown at twenty-four frames per second, how fast was the stage-coach moving?

A/8

I went to pay my monthly account at the local garage last week. When presented with the bill I found I hadn't enough money in my pocket. The proprietor said it didn't matter, because it would pay him to take less.

How is this possible?

A/9

A stranger walked into a public bar, put tenpence on the counter and asked for half a pint of beer. The barmaid asked whether he would like Flowers or I.P.A. The stranger asked for Flowers.

Another complete stranger entered the bar, put tenpence on the counter and asked for half a pint of beer. Upon which the barmaid immediately pulled half of Flowers. How did she know what the second man, who was a stranger to her, wanted?

A/10

The two wheels on each axle of a railway locomotive are rigidly connected together. When an engine negotiates a bend one would expect one of the wheels to skid (because one wheel has to go further than the other).

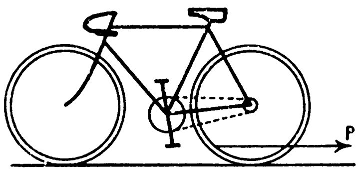
Neither wheel skids, however, even if the rails aren't banked. Why?

A/11

We have two containers, one holding red paint and the other an equal quantity of green paint.

1. One pint of green paint is poured into the red.
2. Two pints of this mixture is poured back into the green.
3. Half the mixture in the green paint tank is poured into the red container.
4. The paints are mixed thoroughly between each operation.
5. If the two containers are then levelled off, without pouring any paint away, so that there is the same quantity in each, which is the more pure, the 'green' paint or the 'red'?

A/12



We show you here a sketch of the authors' bicycle. It is a perfectly good bicycle except that it has a piece of string caught up in the rear wheel. If we pull the string in the direction *P*, will the bicycle move forward, move backward, or 'stay put'?—assuming that the wheel does not slip on the ground.

A/13

All motion is relative. At least that's what they tried to tell us at school. But look at it this way. If motion is relative it is the same thing, of course, to say the Earth is spinning as to say the Earth is fixed and the stars are rotating round it.

But if a body spins it bulges at the equator. In fact the Earth has done so, which is why it has a slightly greater diameter equatorially than from pole to pole. If you consider the Earth as fixed and the stars as rotating round it, you then have to admit that the

effect is to expand the Earth at the equator. And this would be true however small the stars were and however far away they were. But this is a ridiculous state of affairs! What is the explanation?

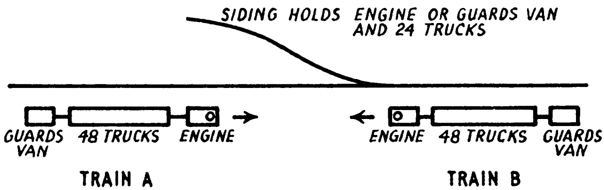
A/14

On a table are two identical bars of soft iron. One has been magnetised. Can you tell which is which? You may move them but not lift them from the table—nor may you use any other object.

A/15

If a girl takes three steps to a man's two steps and they both start out on the left foot, how many steps do they have to take before they are both stepping out on the right foot together?

A/16



How do these trains pass so that each can carry on down the single track in exactly the same order of trucks and van as they are now?

A/17

You have a barrel containing 8 gallons of beer, and two jugs, one that will hold 5 gallons and the other 3 gallons. How do you divide the beer equally between two men (i.e. 4 gallons each) without spilling any or using any other receptacle?

A/18

What is the minimum number of pennies that can be placed upon a table so that each penny touches three, and only three, others? All the pennies must lie flat on the table.

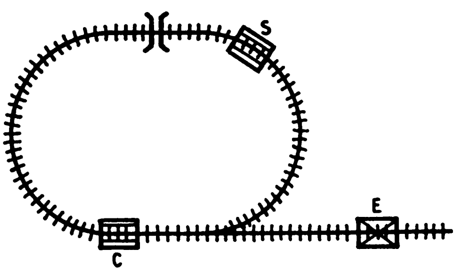
A/19

Everybody knows 0°C is the same as 32°F and that 100°C equals 212°F. But what temperature gives the same reading on both centigrade (Celsius) and Fahrenheit scales? Also, when does the Fahrenheit temperature reading equal the Absolute temperature reading?

A/20

It has been found that many grandfather clocks stop on a Thursday rather than on any other day. Can you offer any explanation for this?

A/21



Here we show a small portion of the British Railways shunting yards at East Wapping (you do appreciate we cannot possibly show you all of it). Only the engine can pass under the bridge, and the problem is to reverse the positions of the truck containing sheep and that containing cattle, returning the engine to its present position.

A/22

The letter *y* can be used as a vowel, as in *spy*, or as a consonant, as in *you*. Can you give (i) a word containing all six vowels once and once only in alphabetical order; (a word like FACETIOUSLY); (ii) a word which has all its letters in alphabetical order?

Which is the longest word you can give in which all consonants are in alphabetical order?

A/23

You should not use a preposition to end a sentence with. Everybody will tell you this is a habit you must get out of. Otherwise we hatc to think of the trouble you will be letting yourself in for. The best thing to do is to get it all out of your system by finding out how many prepositions you can fit in at the end of a sentence which makes sense. We have a solution of nine; can you beat this?

If a word is in some context or other a preposition we will admit it even if it has adverbial force.

A/24

The hour hand and the minute hand on a clock travel at different speeds. There are certain occasions (eleven every twelve hours) when the hands are exactly opposite each other. Can you give a simple formula for calculating the times of these occasions?

Weights and Dates________________________________B

B/1

'On what day was I born?' asked a friend.

'The 28th of August, 1934', we said.

'No, I mean what day of the week?'

'Now let us see, four into 34 goes eight. Eight plus 34 is 42 and as that is a multiple of 7 we can forget about it. August is the eighth month, and the eighth word in the sentence 'O from such a stupid and a silly ad O save me' has 5 letters. 28 less 5 is 23, which divided by 7 leaves 2. The second day of the week starting from Monday is Tuesday. You must have been born on a Tuesday'.

' I don't follow that', said our friend.

We didn't really expect him to! If you go through it carefully, you can, with the hints given, discover the method of finding out what day of the week any given date was or will be (in the present century).

B/2

Here is an old problem worthy of mention, though, if you think it too easy, we do not apologise since it is included by way of introduction to problem number **B/6**.

Given twelve ball bearings, one of which is known to be lighter or heavier than the others, you are asked to locate this odd one and determine its relative weight in three weighings. To accomplish this you are furnished with a balance but no weights.

B/3

I've just been reading Jules Verne's *Around the World in Eighty Days*—you know, where Phileas Fogg lost a day on the way round. Our science master says that ships put it right nowadays by having a thing called a Universal Date Line in the Pacific. When you cross the line from East to West you put the calendar on a day; and when you cross it the other way you put the calendar back. What I want to know is, when Puck put a girdle round the Earth in forty minutes and presumably did the right thing on crossing the Date Line, why didn't he get back on the day before he started—or the day after, according to which way round he went?

I asked the English master this and he got quite cross about it and said it was nothing to do with Shakespeare. But if you flew round the earth as quickly as Puck it would matter, wouldn't it?

PS. What time is it at the North Pole?

B/4

A factory has ten machines, all making flywheels for racing cars. The correct weight for a flywheel is known. One machine starts to produce faulty (over- or underweight) parts.

How, in two weighings, can the faulty machine be found?

B/5

1959

Two days ago I was ten years old; next year I shall be thirteen. What is the date today and when is my birthday?

B/6

There were one hundred and twenty coins in a gas-meter and one of them is either heavier or lighter than the others, but you don't know which. Isolate this coin and tell us whether it is lighter or

heavier in five weighings. (This is a more advanced version of puzzle number **B**/2).

If we merely asked you to isolate the odd coin, regardless of whether it is lighter or heavier, how many coins could you have tackled in five weighings?

B/7

A wholesale merchant may have to weigh amounts from one pound to one hundred and twenty-one pounds, to the nearest pound. To do this, what is the minimum number of weights he requires and how heavy should each weight be?

B/8

A curate is visiting a vicar at his rectory one day and they find that their birthdays are on the same day. The vicar remarks that three of his parishioners have their birthdays on the same day.

The ages of the three parishioners have a product of 2,450 years and added together are equal to twice the curate's age. The vicar asks the curate, 'What then are the ages of the three parishioners?' The curate sat thinking for an hour (for he was not very quick at mental arithmetic) and then he said to the vicar, 'You haven't given me enough information!'

So the vicar said, 'I'm so sorry, I am older than any of my parishioners and am the same age as the product of the two youngest.'

How old is the vicar?

B/9

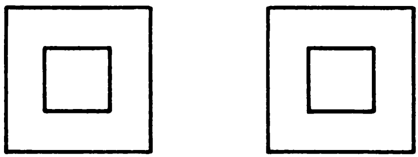
15·3·45·

When I write the date at the head of a letter (as I have written it above to mean the 15th March 1945) I always get a kick out of it when the product of the first two numbers equals the third.

Now which year of the twentieth century gives the greatest number of occasions of this kind?

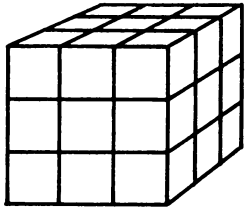
Areas and Shapes — C

C / 1



We have given you two views of a solid object and ask you to draw the third. There are no dotted lines missing. You have the 'plan' and 'end' elevation. What does the side elevation look like?

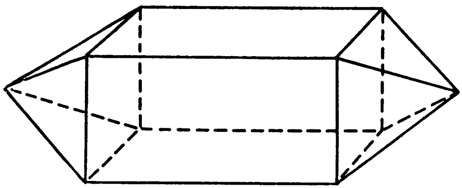
C / 2



A solid cube 3 in. $\times$ 3 in. $\times$ 3 in. may be cut into twenty-seven cubes 1 in. $\times$ 1 in. $\times$ 1 in. by cutting the large cube only six times; i.e., by slicing twice in each of the three mutually perpendicular planes.

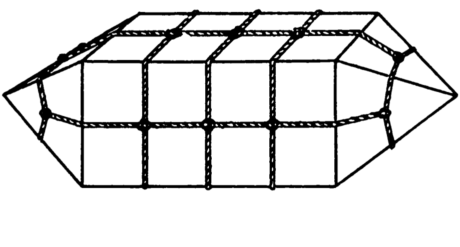
By making one cut and placing the slice formed on top of the remainder before cutting again, is it possible to produce twenty-seven cubes with fewer than six cuts?

We found Peter wrapping up one of the queerest shaped parcels we had ever seen. It was like this.



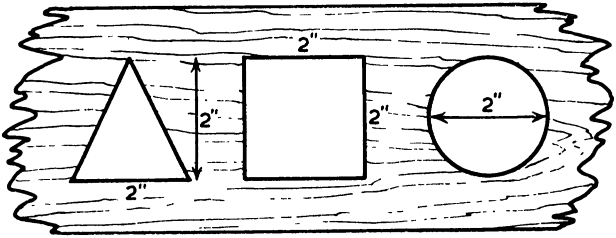
In other words, an ordinary rectangular box with a pyramid on each end.

'What I want to do', explained Peter, 'is to tie it up so that the string goes like this:



(I can only draw part of it, but the sides that you can't see are to be the same). No strands are to be doubled, that is, each knot must only be connected by a single strand of string to the next knot. How many separate pieces of string shall I have to use?

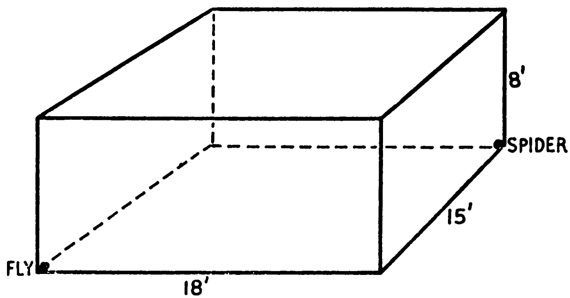
C / 4



Holes of the above shapes were found in a plank of wood.

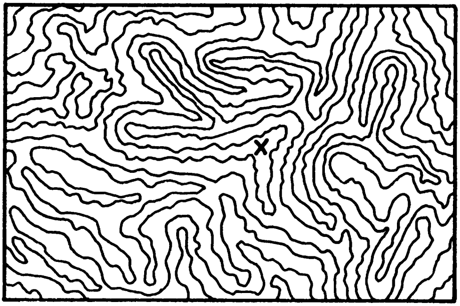
Can you from a 2 in. $\times$2 in. $\times$2 in. cube of wood make a shape (all in one piece) which will pass through each hole and completely fill it ?

C / 5



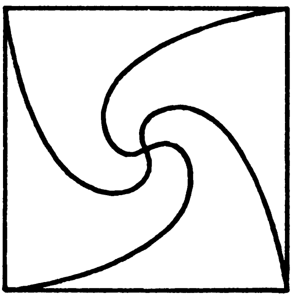
We have drawn a sketch of a room with a spider on the floor in one corner. A fly settles at the diagonally opposite corner and the spider sets out to catch it. Not wishing to get trodden on, the spider takes the shortest route without walking on any part of the floor. If the room is 18 ft by 15 ft by 8 ft, how far does the spider have to walk?

C / 6



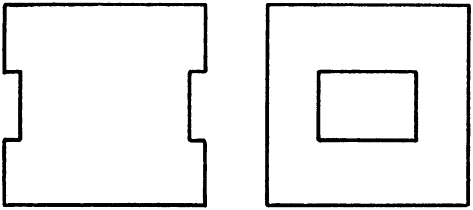
Take a length of string and tie the two free ends together making a loop. Spread the loop over a flat surface in a random manner, but with no part of the loop crossing any other part. Cover all except the centre portion of the maze, so that there is no way of telling whether the point *X* is in or outside the loop. How may any number of *X*'s be drawn on the maze, all of which will lie on the same side of the string (all inside the loop or all outside)?

C / 7



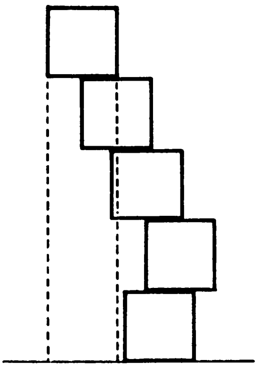
Consider a square field with a man standing at each of its four corners. If each man walks directly towards the man on his right they will all eventually reach the middle of the field together assuming they walk at the same speed. How far will each man have to walk?

C/8



How good are you at visualizing solid objects from blueprints? Draw the third elevation.

C/9

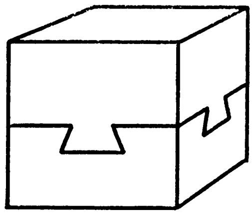


Can you place one cube upon another, upon another, etc., until the plan view of one of them lies entirely outside the area of the base cube?

C/10

A farmer is cutting a field of oats with a machine which takes a 5 ft cut. The field he is cutting is circular and when he has been round it $11\frac{1}{2}$ times (starting from the perimeter) he calculates that he has cut half the area of the field. How large is the field?

C/11

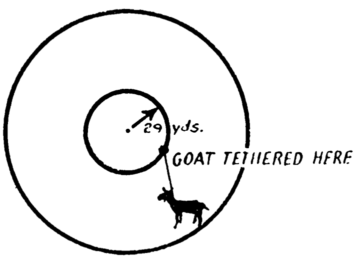


We show an isometric view of two wood blocks that have been dove-tailed together. The other three faces look just the same as the three shown. How was the cube constructed?

C/12

Our table top is circular and its diameter is fifteen times the diameter of our saucers, which are also circular. How many saucers can be placed on the table so that they neither overlap each other nor the edge of the table?

C/13



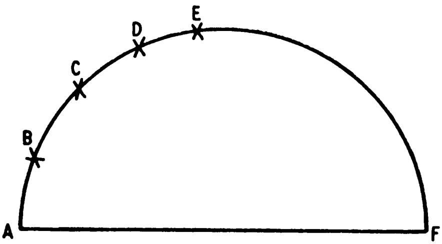
Most puzzle books give at least one problem concerning a goat in a field. We feel that we ought to keep up this tradition. However, our problem is a little more complex than most!

Our great-great- . . . great grandfather was an eccentric who left a considerable fortune to be devoted to the upkeep of his mausoleum. The Will ran thus:—

' . . . a goat must be tethered to the wall of my mausoleum so that it can just keep the wall clear of grass as well as eating half the area of the field. The mausoleum walls must be circular and concentric with the field, also circular, in which my body lies. The remainder of the field . . . '

Now the mausoleum is 58 yd in diameter. How large is the field?

C/14



The diagram shows a picture of the road running from A to F and the semicircular road on AF as diameter. B, C, D, and E are towns on this road and it is remarkable that the distance of each (as the crow flies) from A and from F is an exact number of miles, as also is the distance from A to F. This could not be true for any shorter distance. How far is it from A to F?

Humorous____D

D/1

Among the less important apocrypha of the nineteenth century is the following fragment from the Swiss Family Robinson.

'We went into the inner cave where Francis and Ernest laughingly disclosed the surprise they had in store for us. It was a bicycle which they had built in the long winter months during which we were confined to the cave. I was unable to restrain an exclamation of gratified astonishment at their industry and skill, and the warmth of my commendation brought blushes of pleasure to their cheeks. As for Fritz, his enthusiasm knew no bounds.

'Oh Papa', he said, 'I must be the first to ride this when we emerge from our winter retreat. I will ride all over our domain at furious speeds'. 'Not so fast', rejoined Ernest, 'Since Francis and I have built the machine, ours is the right to use it. But', he added, noting Fritz's crestfallen expression, 'with Papa's permission I shall make a bargain with you. If you can find out how fast the machine will travel without actually riding it I shall let you be the first to essay our new means of locomotion'.

'And how do I do that?' inquired Fritz.

'In the simplest possible way', replied our little professor in his best didactic manner. 'I shall allow you to assume that on the level you will be able to make two complete revolutions of the pedals in one second, and I shall tell you that the circumference of the wheels is three yards. But you may not use any measuring tapes, nor do I tell you the ratio of the gearing from pedal to wheel'.

Our impetuous Fritz immediately clamoured for pencil and paper but could make no progress. 'Then', said Ernest, 'I will show you. If James will kindly give me *A* on his clarinet and my

dear Mamma will lend me a hairpin we shall have no difficulty in ascertaining the speed of the bicycle'.

How did Ernest do it?

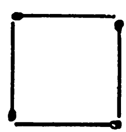
D/2

Two identical trains, at the equator, are travelling round the world in opposite directions. Which will wear out its wheel treads first, assuming they start together, run at the same speed, and are on different tracks?

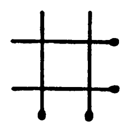
D/3

A certain rather complicated crossing is guarded by two sets of traffic lights both of which have to be crossed to reach the far side. The other day we happened to be passing and saw a bus-driver approach this cross-roads and pass the traffic-lights on red. There was also a policeman on point duty who was signalling traffic to stop. This signal the bus-driver also ignored. At the far side of the cross-roads he stopped on a zebra crossing. How many traffic regulations had he broken?

D/4



If I place four matches in the form of a square they form four right angles.



If I place them thus:—
they form sixteen right angles.

If we remove one, see if twelve right angles can be formed. Don't bend or break the matches!

D/5

A plane had to make a forced landing in the desert because of a faulty engine. Aboard the plane, however, was a mechanic who diagnosed the trouble and needed a 10 thou. feeler gauge to put it right. The only gauges on the plane were a 14 thou. and a 4 thou. gauge.

How did the mechanic obtain the necessary gauge?

D/6

In the London area there are 12,000 double-decker buses and about 1,200 other buses travelling on an average say 200 miles a day each. The fuel consumption of a fully laden double-decker bus may be taken as 20 miles to the gallon, but here is the snag: double-decker buses use one and a half times as much fuel as the other buses and a fully laden bus of any type consumes twice as much fuel as an empty one. Assuming that all the buses are empty for one third of their travelling time and that they work a five-day week, how many gallons of petrol did they consume in January 1959?

D/7

A man sets out from home and walks 10 miles south. He loads his gun and walks 10 miles east. At this point he shoots a bear and returns home by walking 10 miles north.

What colour was the bear?

D/8

UNCANNY INCIDENT IN THE NAWITI-NAWITI ISLANDS

'Seeing you play about with that piece of string' said our Uncle George, 'reminds me of a queer experience I once had while

growing coffee in Nawiti-Nawiti. The natives of those islands, as you know, are incorrigible thieves and I was outraged on one occasion, on returning unexpectedly to my bungalow, to find one of them sneaking out of the back door wearing a complete set of my clothes: shirt, waistcoat, coat, pants, trousers, shoes, and socks. I grabbed him and sent a message to the local Police Commissioner, who, however, was away and was not expected to return until the following morning. To preserve the necessary evidence (for the natives are also incorrigible liars and will perjure themselves white in the face) I locked the fellow up in a room; and to prevent him from removing the clothes I handcuffed his hands together and his feet together.

'Imagine my astonishment, when the Commissioner and I entered the room the following morning, to find this fellow stark naked and asleep in one corner of the room and my clothes neatly folded in the opposite corner.'

'That's an easy one', we said, 'He had merely slipped the handcuffs, removed the clothes, and put the handcuffs back again'.

'On the contrary', said Uncle George triumphantly, 'He had done nothing of the kind. The natives of Nawiti-Nawiti are remarkable for their slender wrists and ankles and their large hands and feet. Although the handcuffs were quite loose on him he could not have slipped them. In fact I learned later that they had never left his hands and feet the whole time'.

'Then an accomplice . . . ' we began.

'Absolutely impossible. He did it entirely by himself. Moreover I examined the clothes and although rather rumpled they were undamaged. I mean, he hadn't picked the seams apart and then sewn them together, or anything of that kind'.

'But that's absurd', we said 'He could not take off a whole suit of clothes with his hands and feet tied together'.

'Well, he did' said Uncle George. 'I ought perhaps to tell you that it was a light silk suit, such as the Europeans wear in those parts. And his hands and feet were not exactly manacled together. The handcuffs consisted of two circlets joined by about two feet of chain, so that he could move his arms and walk about'.

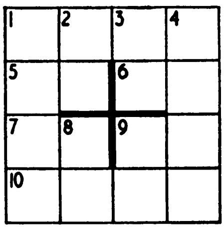
'No witchcraft?' we said suspiciously.

'No witchcraft. An absolutely rational explanation'.

How did the native manage it?

"Cross-numbers"________________________________E

E/1



Here is an easy cross-number problem. Not all the information given in the question is necessary for a precise solution.

Across

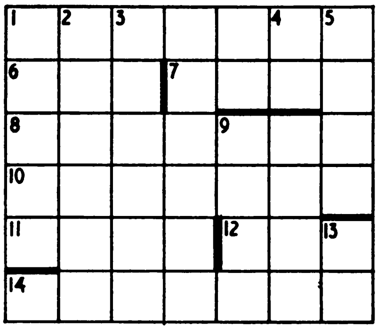
1. $ab^2c - bc - c$
5. $\dfrac{cb}{2}$
6. $bc + 1$
7. $b^2 - bc$
9. $b - 2$
10. $4b^3 - 2$

Down

1. $3ab^2$
2. c^3
3. $\dfrac{b^2 + c}{2}$
4. $b(4b^2 - 1)$
8. $a(2b - 1)$
9. $b - 1$

Find a, b, and c, and complete the cross-number.

E/2



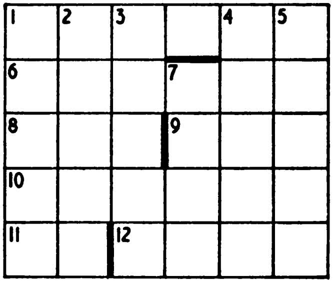
Across

1. There are three circles in this square.
6. (Reversed) Factor of 10 across.
7. Digits in arithmetic progression.
8. A palindrome.
10. Product of 6 reversed and 9 down.
11. A triangular square.*
12. This number plus sum of all the digits in the completed cross-number is equal to a perfect cube.
14. A prime power of a prime number.

Down

1. If you divide this by any number from 2 to 12 inclusive the remainder is 1.
2. Product of two numbers differing by 2.
3. No digit occurs more than once in this column.
4. (Reversed) Sum of factors of 5 down including unity.
5. (Reversed) The smallest integer expressible as the sum of two cubes in two different ways.
9. Factor of 10 across.
13. The number of which 14 across is the power.

*NOTE: A triangular square is a number of the form $\frac{1}{2}n(n+1)$ for instance 45.

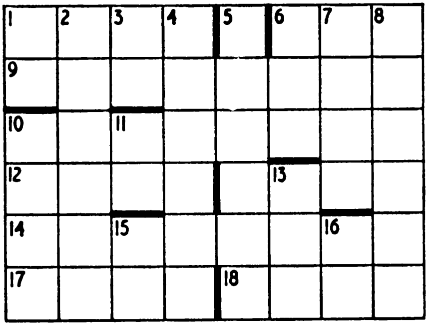


Across

1. The second three form a number equal to three times the first three plus three.
6. To multiply by two put the first two digits at the end.
8. A square palindrome.
9. The sum of the digits is three.
10. The first and second, third and fourth, fifth and sixth digits add up to the same total.
11. A multiple of the sum of its digits.
12. Cube of a prime plus square of another prime.

Down

1. Fourth power.
2. Power of two.
3. Reverse power of three.
4. Contains a digit twice which which does not appear elsewhere.
5. 7 down in the scale of 7.
7. See 5 down.



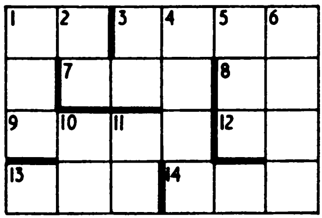
Across

1. A square backwards and forwards.
6. The nine squares of which this is the top L.H. corner form a magic square.
9. Reminiscent of blackbirds.
10. No prime factor over twenty.
12. Prime to 10 across.
14. To get practice in writing a digit multiply this by five times the digit.
17. Shahrazad's (or Scheherazade's) favourite number.
18. See one down.

Down

1. Sum of prime factors of 18 across plus 240 (excluding unity).
2. Eve ate an apple and Adam this . . . .blige Eve.
3. Found in the subject of 17 across.
4. Prime to 10 across.
5. (Reversed) With 16 down at end equals all but five digits of a power one third of 8 down summed.
7. Arithmetic progression of digits.
8. Multiple of 17 across.
10. A prime and so are the first three digits, the middle two and the sum of the first two, but the last three are not.
11. Sometimes round.
13. This is a beast.
15. Number of edges possessed by one of the regular polyhedra.
16. See 5 down.

E/5

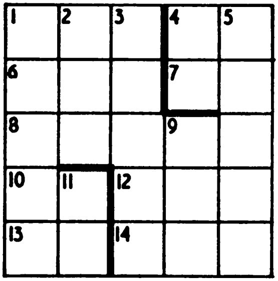


Across

1. A multiple of the sum of its digits.
3. Divisible by 12 reversed with remainder 8 reversed.
7. The square of any number ending with these digits terminates in three fours.
8. See 3 across.
9. A prime multiplied by ten.
12. See 3 across.
13. Square sum of digits of 1 down.
14. Sum of greatest primes in 1, 7, 9, 13 across, 1 (reversed), 2, 10 down.

Down

1. Square of sum of digits of 13 across (reversed).
2. Number of distinct damnations.
3. Same as 10 down.
4. The sum of the first three digits of this square equals the last. Reverse it.
5. A muddle of 1 down.
6. Product of 3 down and 14 across.
10. Not a square but the sum of two squares.
11. Just short of a century.



Across

1. b^2
4. $a+c-1$
6. c^4-d^2-2a
7. $ad^{\frac{1}{2}}$
8. $2a^2b^2$
10. b
12. b^3-a^3
13. a^2
14. a^3

Down

1. $3cb^2d-10\,b^2-11$
2. $abd^{\frac{1}{2}}$
3. $2cb^2d+3$
4. $\dfrac{c^2-1}{2}$
5. $2a^2b^2+c^2$
9. $2c^2+a^2-d$
11. $a+b+1$

Find a, b, c, and d.

Analytical____F

F/1

A rope ran over a pulley; at one end was a monkey, at the other end a weight. The two remained in equilibrium. The weight of the rope was 4 oz per ft, and the ages of the monkey and the monkey's mother amounted to four years. The weight of the monkey was as many pounds as the monkey's mother was years old, and the weight of the weight and the weight of the rope were together half as much again as the weight of the monkey.

The weight of the weight exceeded the weight of the rope by as many pounds as the monkey was years old when the monkey's mother was twice as old as the monkey's brother was when the monkey's mother was half as old as the monkey's brother will be when the monkey's brother is three times as old as the monkey's mother was when the monkey's mother was three times as old as the monkey was in paragraph 1.

The monkey's mother was twice as old as the monkey was when the monkey's mother was half as old as the monkey will be when the monkey is three times as old as the monkey's mother was when the monkey's mother was three times as old as the monkey was in paragraph 1.

The age of the monkey's mother exceeded the age of the monkey's brother by the same amount as the age of the monkey's brother exceeded the age of the monkey.

What was the length of the rope?

F/2

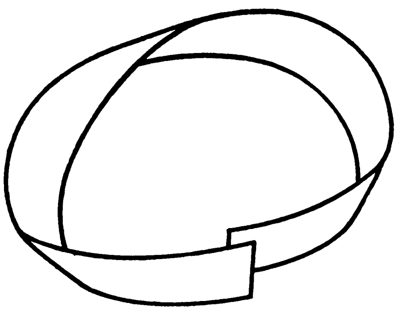
Our club dinner was a great success except that I, as secretary, had to see that everybody paid his bill afterwards. The meal

was the same price for everybody, so there was no difficulty about that. But the drinks were a dreadful nuisance. We had water, beer, and wine to drink, and it was arranged that anyone who drank water should pay one shilling, beer two shillings, and wine five shillings; so that, for instance, if a member drank water and beer he paid three shillings, and if he drank all three he paid eight shillings. Not being able to keep track myself of what everybody drank (for I had to write the chairman's speech for him) I asked some of the others to do it.

Unfortunately they made a frightful mess of it and all the information I have been able to salvage is this: the number of people who didn't drink water was 18. The number who drank wine was 39. The head waiter told me that 106 glasses were used only once and the total amount payable for drinks was £14 13*s*. 0*d*. I also happen to know that there were 9 teetotallers present.

The irritating thing is that if all the people who drank wine only had each counted a particular class of drinkers, I might have been able to say exactly how many people drank what. As it is, I can't, but assuming that the number of people who drank beer and water, but not wine, was as great as it could have been, how many people at least drank all three?

F/3



Take a rectangular strip of paper, give it a twist and stick the ends together.

This is one of the most remarkable surfaces known to geometry. It has only one side because you can get to any point from any other by a path which doesn't leave the surface (and does not cross the edge).

Suppose you take a pair of scissors and cut along the middle of the strip, all the way round, what will happen?

When you have done this, try and think what would happen if you gave the strip two twists before joining the ends and cutting it down the middle. Before you make a model, try it in your head!

F /4

A ship is twice as old as its boiler was when the ship was as old as the boiler is now. The combined age of the ship and the boiler is thirty years. What is the age of the ship and of the boiler?

F /5

The positions of the eleven players who make up a soccer team are:—

Forwards.	Outside left	Inside left	Centre	Inside right	Outside right
Half-backs.		Left	Centre	Right	
Full-backs.		Left		Right	
			Goalkeeper		

Below are 15 statements about the players of a certain soccer team. From these statements it is possible to deduce what position in the team each player fills.

(*a*) James, Parks and Dale, as well as the inside right and outside right, are bachelors.
(*b*) Smith and the three half-backs play golf together in their free time.
(*c*) Swift is taller than any of the five forwards but he is shorter than the right full-back.
(*d*) Burns dislikes the left half-back.
(*e*) The centre-half is captain of the team.

(*f*) Gardiner, Jones, and the two full-backs have all had tempting offers to go to Italy; but Dale, who has been there, has persuaded all four of them to turn the offers down.
(*g*) The goalkeeper and the centre-forward each have two children by their present wives.
(*h*) Swift has been married longer than either of the two married half-backs; the left half-back was married a week ago.
(*j*) The centre half-back is divorcing his wife.
(*k*) Gardiner and Rakes are not forwards; Rakes is vice-captain of the team.
(*l*) The left full-back is engaged to Swift's sister; Dale and the right half-back are each engaged to be married. Meanwhile Dale is staying with the inside left and his wife.
(*m*) Smith and Evans both have wives who are good cooks.
(*n*) James and Parks never play in a left position; James is a better kicker than the right half-back.
(*o*) The captain has to keep an eye on Burns and the five forwards who all tend to drink too much.
(*p*) Evans has scored more goals than the centre-forward and Robinson has scored more than the outside right.

F/6

A man is rowing across a river in the gathering dusk. On the approaching bank he sees the dim figures of three men. 'Some' of these men wear black and 'some' white, and the oarsman wishes to know which wear black and which wear white, so he calls out,

'What colour are you?'

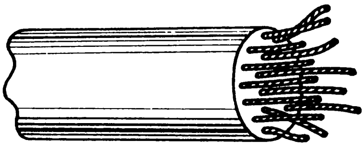
The reply from the first dim figure is lost in the slight breeze, but the second man calls out,

'He's black, but I am white!'

The third figure, referring to the first man, says,

'He says he's white and he is white!'

With this information, and with the knowledge that the men wearing black always lie, and the white dressed men tell the truth, the man in the boat identified the figures for what they really were.



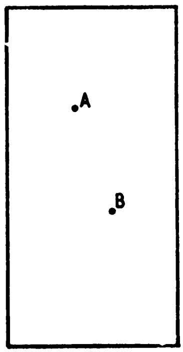
A multicored cable is laid in a ditch ten miles long. Unfortunately it was not made with multi-coloured wire and the 'ends' need to be identified.

What is the least number of journeys that one man must make up and down the cable to tag all the wires?

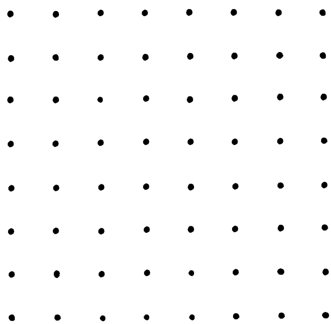
The only equipment he has is a six-volt battery, a six-volt bulb, and some spare wire (say six yards of it).

Assume that he starts at one end of the cable and finishes at the other end with the cable ready for use.

Cards and Games______________________________G



A friend has been showing us how to play billiards, but we can't get the shots off the cushions right. For example, here is a position we got to yesterday. What we wanted to do was to hit the opponent's ball, which was at *A*, with our ball at *B*, by a shot rebounding from all four cushions. We had no idea in which direction to hit our ball, but felt there must be some kind of rule to help us. Everybody we ask says it is done by instinct and practice; what do you think?



We show a rectangle of dots that can be made into a very simple game. It may be of any convenient size but, in order to avoid drawn games, the number of dots in any row or any column is made even.

Two players play alternately, a move consisting of joining two adjacent dots horizontally or vertically with a straight line. The object is to complete as many squares as possible, and each time a player completes a square he initials it and has another move. The player who completes the greatest number of squares wins.

(*a*) Which player should always win, first or second?
(*b*) How should he play to win?

G/3

Tiddly-Winks League

	Matches				*Games*		*League Points*
	Played	Won	Drawn	Lost	For	Against	
King Alfreds House				2	111	57	34
Goodlake Arms				3	104	64	32
Castle				2	93	67	30
Grain				4	99	61	29
Lamb				1	91	53	27
Hare				10	87	89	20
Fox and Hounds				9	82	94	20
Buckland				10	75	93	18
Boars Head				11	81	87	17
Prince of Wales				12	63	89	13
Bell				12	70	98	12
Star				14	67	101	9
Sparrow				14	49	111	7
Malt Shovel				16	47	105	3

League points awarded as follows:— 2 for wins
1 for draws

How many games constitute a match?
Which team has drawn the most games?

G/4

KH	9C	4D	QD	6H	7S	5H	4H	10C
7D	JC	AS	10S	9H	7C	AD	9D	
3S	6D	AH	KC	QS	3C	6S		
2S	5D	KS	4C	2C	8H			
KD	AC	8C	JS	7H				
5S	8S	JD	10D					
9S	QC	JH						
6C	4S							
5C								

QH, 10H, 8D, 3D, 3H,
2D, 2H.

This is a standard form of Patience called King Albert. One

card may be moved at a time and when the aces are uncovered they should be removed and built upon A—2—3 etc. How many moves do you need and what are they?

Red cards may not be placed upon red cards, neither may black be placed upon black. For example, the six of spades may be placed upon the seven of hearts which may be placed upon eight of clubs or spades etc.

In King Albert Patience any card may be promoted to a blank file, not merely the kings. The effect of this is that when a file is blank two consecutive cards may be moved at once; when two files are blank four may be moved at once, and so on.

G/5

4D	6C	10D	5S	3D	10C	5H	AD	8S
JD	QD	2S	3H	9D	9C	3C	5D	
AH	QC	2H	AC	4C	5C	2D		
AS	KH	8C	JS	7H	JC			
8D	10S	3S	7D	4H				
7C	2C	10H	KC					
8H	6D	7S						
QH	9S							
6H								

KD, KS, QS, JH, 9H, 4S, 6S.

G/6

AH	2H	AD	5D	5H	QH	8C	4D	10D
KS	3H	JD	QS	2C	AC	10H	QD	
6C	JS	8D	4H	QC	7H	KH		
9S	5S	6D	10C	7C	10S			
3C	AS	7D	4S	9D				
6S	KC	JH	7S					
8S	JC	8H						
3D	9C							
6H								

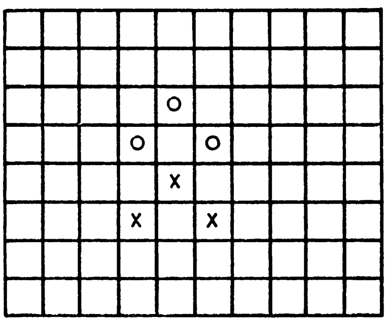
KD, 2D, 2S, 3S, 4C, 5C, 9H.

G/7

JD	6H	AS	KH	7S	3S	9S	7C	JC
9D	8D	10S	2H	3H	KC	JH	10H	
8S	8H	9C	6D	5S	5D	10C		
7D	5H	AC	6C	QD	6S			
2S	AH	5C	2D	KS				
3D	KD	3C	4H					
AD	QC	10D						
9H	2C							
4S								

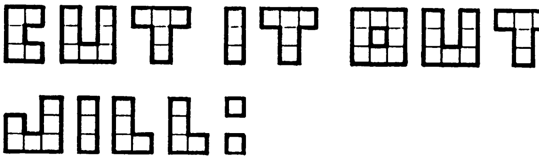
QH, QS, 4C, 8C, 4D, JS, 7H.

G/8



This is part of a game called Peggotty. The object of the game is get five in a row with players taking it in turns to play, just as in noughts and crosses. The size of the board is unlimited. If Nought is to play next, how should he win?

G/9



Can you cut a chess board so that it forms the statement above?

G/10

We found Aunt Agatha shuffling a pack of cards in what she calls 'the good old-fashioned way'. She holds the pack in the left hand and transfers it one by one to the right hand by taking one card from the top to begin with, then another card from the top of the left-hand pack to put on the top of the right-hand pack, then one from the top of the left-hand pack to the bottom of the right-hand pack, then one from the top of the left-hand to the top of the right-hand, and so on, cards coming from the top of the left-hand pack all the time and going alternately to the top and bottom of the pack which is being built up in the right hand. Peter, fascinated by the meticulous way in which she did it, stood open-mouthed.

'Did you know', I asked, 'that if you keep on shuffling the pack over and over again like that you get back to the original order?'

'I suppose you do eventually', replied Aunt Agatha, 'but not until you have done a lot of shuffling'.

'Not so much as you seem to think', I said, 'In point of fact 51 shuffles will do it for an ordinary pack of 52 cards. For a pack of 50 cards you would only need 15 shuffles'.

Peter took a pack and began to shuffle. Before very long he said 'But I've got back to my original order in 10 shuffles'.

'In that case there must be some cards missing'.

'There can't be very many or I should have noticed it from the size of the pack', said Peter, but he counted them to make sure.

How many cards were missing from the pack?

'Aunt Agatha's method of shuffling', said Uncle George, 'is far inferior to my own. I just divide the pack into two halves and flick them together like this.'

He took a full pack of cards, divided them into two equal heaps and shuffled them in what is sometimes called the French shuffle. We examined the shuffled pack and found that the top card of the original pack remained at the top and the others were perfectly interleaved so that, for instance, the 27th from the top of the original pack became 2nd from the top, the original 2nd from the top became 3rd, the 28th came 4th and so on.

'I'm not sure that your method wouldn't lead back to the original arrangement quicker than Aunt Agatha's', I said.

'Let's try it', said Uncle George. He took an even number of cards from the pack and began to shuffle. Peter took the remainder and began to shuffle in the same way. Both took care that the interleaving should be perfect. When they had both returned to their starting orders they announced the number of shuffles.

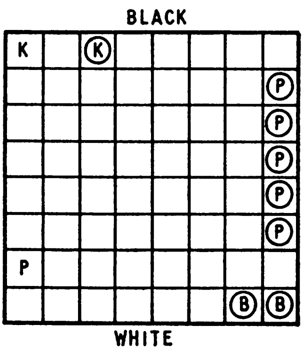
'That's funny', said Peter. 'The number of shuffles I required is the same as the number of cards you took.'

How many cards did Uncle George take?

Chess H

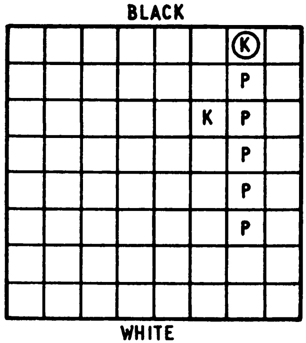
Throughout this section: black pieces are denoted by ringed letters, and white pieces by unringed ones.

H/1



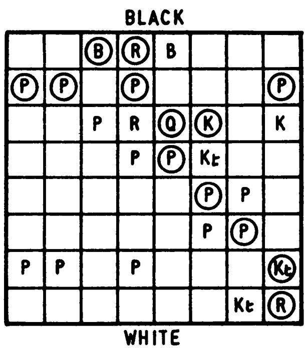
How did Black mate?

H/2



White to play and win.

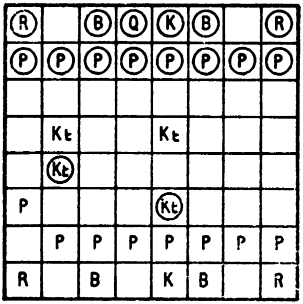
H/3



Who mates? In how many moves?

H/4

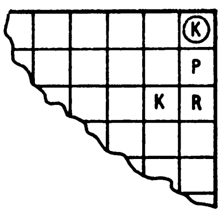
BLACK



WHITE

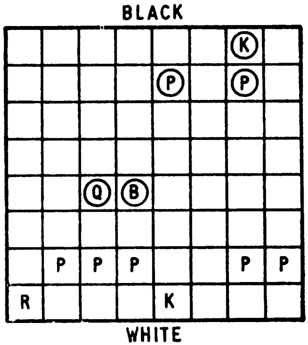
At this point Black played Knight (Kt 5), took Pawn, and announced mate. What was White's reply?

H/5



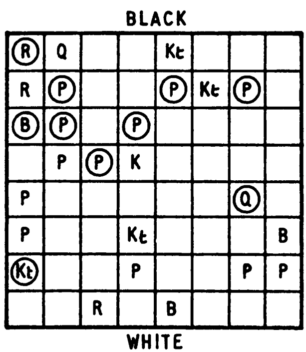
We found the above fragment of a chess puzzle among our papers and we know that it was White's move and that he mated in three. We are not sure which way the board should be, i.e. we don't know which were the black squares and which the white. Can you tell us? How, in fact, did White mate?

H/6

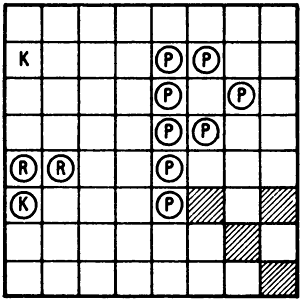


At this point White castled on the Queen's side. Black thereupon mated on the move. How?

H/7



Black, about to be mated in one, removed his King from the board in a fit of pique. Where was it?



How did Black mate?

Probability J

J/1

Struldbrugs

The discovery of an unpublished manuscript of Lemuel Gulliver's throws some new light on a question of Laputan population.

'Towards the end of my visit to Laputa', says Gulliver, 'I allowed myself the liberty of pointing out to the King that if the struldbrugs were permitted to live for ever, being immune from death except by execution and having their numbers continually increased by fresh births, the time must inevitably come when there was no space left on the island for ordinary mortals, so that the race would die out and be replaced entirely by struldbrugs. His Majesty was much struck by this prognostication, which he was pleased to regard as a calamity, and asked whether I had any proposal for avoiding it. I submitted to him that the struldbrugs should be put to death on reaching the age of three score years and ten.

'His Majesty thereupon went into a fit of meditation lasting for three days, and on emerging from it issued a proclamation which admirably illustrates the mathematical subtlety of his mind. It was enacted that whenever a struldbrug was born a scroll should be prepared for him and retained in the Hall of Records. On the struldbrug's day of birth and every subsequent birthday the Court Mathematician should choose at random a letter from the Laputan alphabet and write it on the scroll. When the scroll contained every letter of the alphabet the struldbrug would be quietly put to death. As some compensation it was enacted that a struldbrug could not be put to death for any other reason.

'I was informed by the Court Mathematician himself that the effect of this decree was to make the expectation of a struldbrug's life between 70 and 71 years. Thus the King attained his object

without incurring the odium of sentencing any particular subject to death at a specified time, or in any way violating the principles of humanity and statesmanship for which he is so justly famous'.

How many letters are there in the Laputan alphabet?

J/2

Letter from School

Dear Daddy

I hope you are well. I have run out of pocket-money. There are two chaps here who are up for election to be head of the Black Hand Gang: Chuck and Lefty. They have been round to us finding out how many votes they will get, and nine of us are going to vote for Chuck and six for Lefty. So Chuck will be elected but we still have to have an election and what is worrying him is this: it is one of the rules of the Gang that we have to vote one at a time and the votes are counted as we go along. The order of voting is random and every time Chuck *loses the lead* (even if he does it on the first vote) he has to stand everybody an ice-cream. What is Chuck's chance of getting away without having to buy any ice-cream?

Peter.

PS. Chuck and Lefty cannot vote themselves.

J/3

'Considering that there are only 365 possible days to go round among millions of people', said Peter, 'it is rather funny that one doesn't meet more people whose birthday is on the same day as one's own'.

'That reminds me', I said, 'How many people are coming to your birthday party?'

Peter told me.

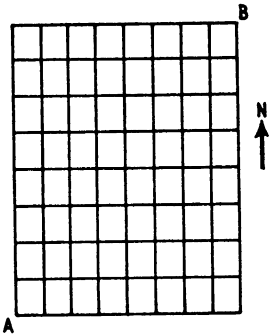
'In that case', I said, 'The chances are that two of them at least will have the same birthday'.

'What abour leap-years?' asked Peter.

'Let's ignore them and make it rather easier'.

How many people at least were to attend the birthday party?

J/4



'This is a map of the roads from *A* to *B*. Every line is a road. If I start to walk from *A* to *B* and you start to walk from *B* to *A* at the same speed, what is the probability that we shall meet?'

'You can take any route you like?'

'Yes, except that you mustn't go back along a path, or away from your destination. For instance, if you come from *B* to *A* you must always move from East to West or North to South'.

'Can you say, what is the probability of our meeting if I cycle and hence travel three times as fast as you?'

J/5

In the Local we found the Landlord throwing a number of dice simultaneously.

'I am trying to get one of each of the six faces', he said, 'But it hasn't happened yet'.

'No', we said, 'You must have at least four more to make the odds in favour of such a thing'.

How many dice had the Landlord?

J/6

Able, Baker, and Charlie engage in a triangular revolver duel. Able can always hit his man and Baker is a better shot than Charlie. It is therefore decided that they shall fire in the order: Charlie—Baker—Able and continue until only one of them is unhit, the turn of a hit man being taken by the next on the list.

Clearly Charlie will not fire at Baker with his first shot, but it may not be to his interest to hit Able either. If all one knows is that there is uncertainty on this point, how nearly can one calculate the probability of Baker's hitting his man?

J/7

There is something strange going on and we can't get to the bottom of it. Suppose we go out and collect some conkers and arrive back with a bagful. Surely it is an even chance whether we have an even number or an odd number? And if we take a handful out of the bag and there was an even number in the bag it is an even chance whether we get an odd or an even number. But if there was an odd number in the bag the chances are more in favour of getting an odd number in the handful because there is one more way of choosing an odd number than an even number. So on the whole the chances are slightly in favour of getting an odd number in the handful. How can this be if the handful is chosen at random like the original bagful?

Arithmetical__________________________________**K**

K/1

'What have you been doing at school today?' we asked.

'Sums', replied Peter, 'The master writes down a number and we have to find what numbers divide into it. Factors, you know'.

'And was it easy?' we enquired.

'Sometimes', said Peter. 'You can tell whether a number divides by two, of course, because if so it must be even. And if the digits add up to a multiple of 3 then 3 must be a divisor. There's something about 11, too. You add up all the digits in the even places and then those in the odd and if the result is the same the number divides by 11. But I get stuck over some numbers. For instance we had to try and factorise 1064893. You can see that 2, 3, 5 and 9 are not factors, but I couldn't do it all the same'.

We glanced at the number and did a few subtractions.

'Nor', we said, 'does it divide by 7, 11 or 13'.

How did we know?

K/2

'I've just been looking round the orchard', said Peter, 'to find how many rows of three trees we have—three in a dead straight line I mean. Actually there are eighteen'.

'But we've only twelve trees in the orchard', I replied.

'I know that but there are eighteen rows of three all the same', repeated Peter.

Can you draw a possible layout of the trees in our orchard?

K/3

```
×××) ××××××××( ×7×××
     ××××
     ----
       ×××
       ×××
       ---
       ××××
        ×××
       -----
         ××××
         ××××
         ----
         ----
```

Only one number is given in this long division sum. We should like you to complete the remainder.

K/4

The following is the most difficult puzzle we know of the type of the previous puzzle. It has the remarkable property that no digit in the working is given.

```
××××××) ×××××××( ××××××××××××
        ××××××
        -------
        ×××××××
         ××××××
         -------
         ×××××××
         ×××××××
         --------
          ×××××××
           ××××××
          --------
           ×××××××
           ×××××××
           --------
            ×××××××
            ×××××××
            --------
             ×××××××
             ×××××××
             --------
              ×××××××
              ×××××××
              ---------
                   ××××××
                   ××××××
                   ------
                   ××××××
```

There will of course be a decimal point in the answer, the last nine digits of which form a repeating decimal.

K/5

'Talking of writing down consecutive numbers', said a friend, 'How many consecutive numbers can you find without a prime amongst them?'

'As many as you like', we said.

'Well, say a dozen', said our friend.

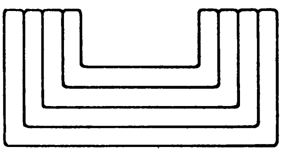
We showed him how it was done.

'And,' said our friend, 'How many consecutive numbers can you find which are all prime?'

How does one write down consecutive series of non-prime numbers of any desired length?

What is the greatest sequence of consecutive numbers which are all prime?

K/6



Four close-fitting, cylindrical containers stand one inside the other. They are all in the same proportion; (i.e. the ratio of internal height to internal diameter is the same for each). The walls are all of the same thickness.

The volume of the three smallest containers equals the volume of the largest container, which has an internal diameter of one foot.

What are the dimensions of the other containers, and how thick are the walls?

K/7

I was trying to make a number plate the other day with those separate digits that you fasten on a plate. It was a four-figure number I wanted. I hadn't the figures 9, 7, and 0, although I had one each of the others. How many four-figured numbers could I have made?

I could of course have used the 6 as a 9 by turning it upside down.

K/8

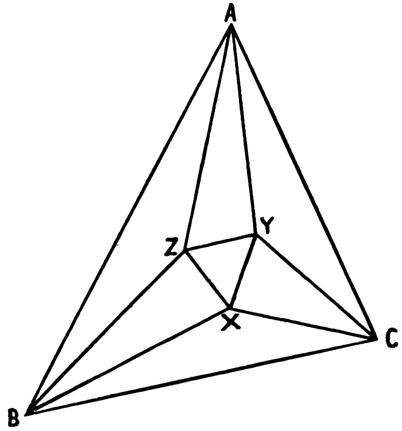
Take three consecutive single digit numbers (single for convenience only) and cube them. Now add up the figures in the three results. Add again if these numbers are not single ones and keep on adding until you are left with just three single digits. Now arrange them in order of magnitude and the figure you now have is 189. Why?

K/9

Here is a square root sum but we copied it out in such a hurry we can't now read any of the figures! All we know is that the number whose square root is to be found has a nine in it somewhere.

```
             ) × × × × × × ( × × ×
               × ×
               ---
××                × × ×
                  × × ×
                  -----
× × ×                  × × ×
                       × × ×
                       -----
                       - - -
                       -----
```

What are the other numbers?



A B C is any triangle. If angles $A\hat{B}C$, $B\hat{C}A$, and $C\hat{A}B$ are trisected prove that the triangle X Y Z is equilateral.

General

Solutions____________________________________A/S

A1 /S

TABernacle 2463 may be written VACCINE. VINcent 8225 may be written as TINTACK.

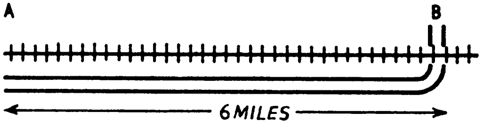
A2 /S

It is not a matter of accounting for 30*s*. In all, 27*s*. has been spent by the three men: 25*s*. went on wine and 2*s*. went to the waitress.

A3 /S

Oidium has a syllable-to-letter ratio of 1 : 1·5, four vowels and two consonants.
Acuity likewise has a ratio of 1 : 1·5, also having four vowels and two consonants.

A4 /S



The man and the train normally meet at the crossing at, say, 8 a.m.

Usual time of man at B is 8 a.m. and at A is 7.30 a.m. When the man is late, he arrives at B at 8.25 a.m. and at A at 7.55 a.m.

Hence the train takes 5 minutes to travel from A to B. Therefore its speed is 72 m.p.h.

A5 /S

The moments of inertia will be different. If both cylinders are rolled along the ground, the leaden one will roll the farther.

A6 /S

The barge always displaces its own weight of water and if the ice were off-loaded into the water it would also displace its own weight of water, the barge then displacing an equal weight less. The water level would remain constant. A block of ice always floats with one-eighth of its volume out of the water, but ice shrinks when it melts. Since it must displace its own weight of water when it melts it must itself be that volume it has displaced. The water level in the lock will not alter.

The sea level will not alter because of the reasons given above. If, however, all the ice at the North Pole were to melt there would be a significant rise in 'sea level' because of the ice sheet that covers Greenland and other arctic land masses.

A7 /S

In $\frac{1}{24}$th of a second the wheel must move $n/12$ revolutions (where n is an integer), i.e., it is revolving at $120n$ r.p.m.

$$\text{Hence speed of coach} = \frac{120n \times 60 \times \pi}{1760} \text{ m.p.h.}$$

$$= 12 \cdot 85n \text{ m.p.h.}$$

The most reasonable stage-coach speed is obtained when $n=2$, giving $25 \cdot 75$ m.p.h.

A8/S

The bill was £2 and I asked for a receipt.

I had £1 19*s.* 11*d.* in my pocket; by accepting this the garage owner saved himself the need of using a 2*d.* receipt stamp. He thus increased his profit by 1*d.*

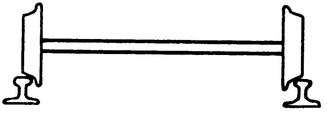
A9/S

Flowers must cost 10*d.* whilst I.P.A. costs 9½*d.*

The first stranger put a sixpence, a threepenny piece, and a penny on the counter.

The second man put a sixpence, a threepenny piece, and two halfpennies on the counter.

A10/S



The wheel treads are coned so that the outer wheel presents a greater circumference when it is pressed outward by centrifugal force, and the inner wheel a smaller one.

A11/S

They will have the same purity.

Whatever percentage of green there is in one container, there will be a similar percentage of red in the other.

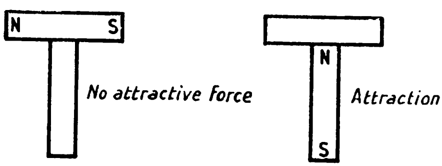
A12/S

The bicycle will move backward.

A13/S

The stars have very little to do with this question. If one were to consider people on a planet where no stars were visible, the effect of spinning the planet would to them seem extraordinary. Without any apparent reason the planet would suddenly bulge at the equator. It is in fact possible to have a world spinning without the motion being relative to anything else in space. We agree that the motion of the stars across the sky is relative but the centrifugal effects within the earth concern only the earth. Putting it another way, spin results in acceleration, since the individual particles do not move in straight lines; and hence is detectable either as centrifugal force or distortion of geometrical shape.

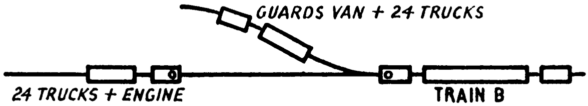
A14/S

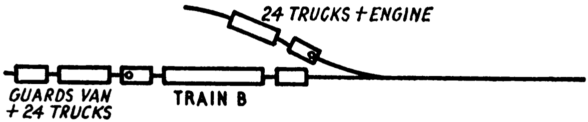


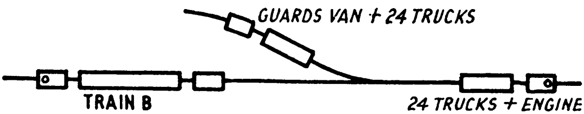
A15/S

They will never step out with right feet together!

A16/S



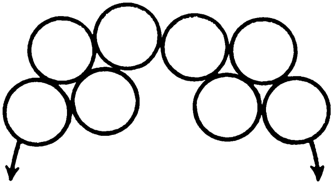




A17/S

8-gall. barrel	*5-gall. jug*	*3-gall. jug*
8	—	—
3	5	—
3	2	3
6	2	—
6	—	2
1	5	2
1	4	3
4	4	—

A18 /S



As shown, the pennies must be made up into units of four and the problem merely consists in how few of these units can be formed into a circle. In fact, five units of four have to be used—giving an answer of twenty pennies.

A19 /S

Let x be the temperature

$$\frac{9x}{5} = x - 32$$

$$\text{i.e.}\quad \frac{4x}{5} = -32$$

$$\text{giving } x = -40$$

$$\text{i.e.}\quad -40°\text{C} = -40°\text{F}.$$

$$(A-273)\frac{9}{5} = A - 32$$

$$4A = 2297$$

$$A = 574{\cdot}25°\text{K}$$

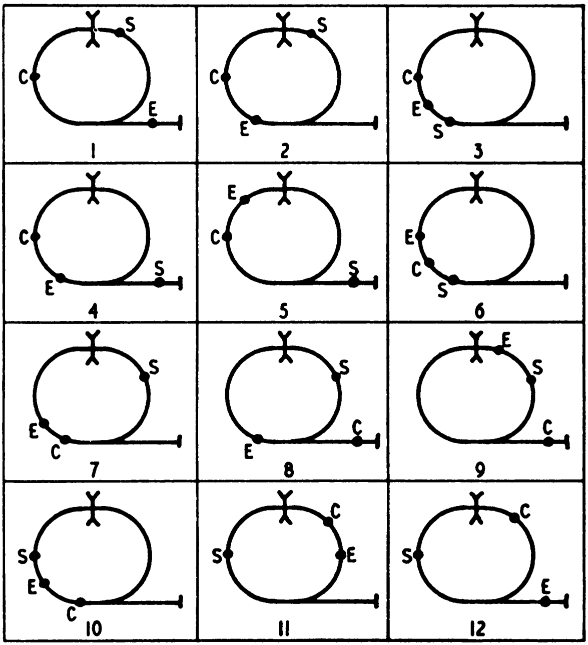
A20 /S

Most grandfather clocks are eight-day clocks. Many are wound on a Sunday or at any rate over the week-end.

Round about Thursday the length of weights becomes similar to that of the pendulum.

Either because the pendulum support is not rigid or because of air currents within the case, the weights begin to swing slightly in time with the pendulum. But the length of the weights is continually changing and the period of swing increases. The 'off-beat' swinging is then quite often sufficient to stop the pendulum and hence the clock.

A21 /S



A22 /S

Abstemiously has the vowels in the correct order.
Billowy has all its letters in the correct order.
Baccalaureate has its consonants in the correct order.

A23/S

There was once an Australian millionaire who used to read to his small daughter from a book of fairy tales. Once on a visit to England she missed this so much that he used to ring her up from Australia and read a piece to her every night. Then he lost the book so she bought another copy in England to send to Australia and her mother enquired 'What did you choose Sunday to send that book that you like to be read to out of from down under off by for?'

(Correspondence about the adverbial force of prepositional forms is discouraged).

A24/S

The following formula gives the minutes past twelve to which the hour hand points when the minute hand is exactly thirty minutes ahead.

$$\text{Minutes past twelve } y = \frac{30}{11}\Big[(n-1)\,2+1\Big]$$

where n is the next hour.

Example. At what time between 4 and 5 will the hands be opposite each other? ($n=5$)

$$\therefore y = \frac{30}{11} \times 9 = \frac{270}{11} + 24\tfrac{6}{11}$$

i.e. the hour hand will be $24\tfrac{6}{11}$ minutes past 4.

The formula may easily be derived from the following.

If x is distance moved by minute hand
y „ „ „ „ hour „
then $x-y=30$.

First time the hands move round	$x=12y$
Second „ „ „ „ „	$x=12y-5$
Third „ „ „ „ „	$x=12y-10$
	etc.

Weights and Dates

Solutions______________________________B/S

B1 /S

January 1st 1900 was a Monday.

January 1st 1901 was a Tuesday.

January 1st in two successive years moves one day of the week, except after a leap year, when it moves two days.

Any number divisible by the number of days in a week can be ignored.

January 1st 1934 was $34+34/4=42$ (Divisible by seven) which was a Monday.

The sentence with twelve words is merely a mnemonic giving the number by which the days from the 1st January to the 1st of each month exceed a multiple of seven.

B2 /S

Number the balls 1 to 12.

Weigh 1, 2, 3, 4 against 5, 6, 7, 8.

If they balance, the odd one must be amongst 9, 10, 11, 12 and we have two weighings in which to locate it. We also know that balls 1 to 8 are perfect ones.

Weigh 9, 10, 1 against 11, 2, 3.

If they balance, 12 must be the odd one, and if it is weighed against a perfect ball we can tell whether it is heavier or lighter.

If they do not balance, note which side is the heavier......(A)

Now weigh 9 against 10..........................(B)

If they balance, 11 is the imperfect one and 9 and 10 are perfect. (A) tells us whether it is heavier or lighter.

If 9 and 10 do not balance, 11 must be a perfect specimen and

we know whether the odd one is light or heavy from (A). The actual ball may now be located from (B).

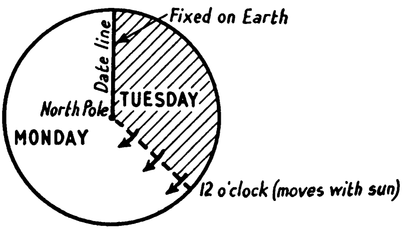
If the first weighing 1, 2, 3, 4 against 5, 6, 7, 8 does not balance, we know that 9, 10, 11, 12 are perfect balls.

Note which side is the heavier........................(C)
Weigh 1, 9, 5, against 6, 7, 2.

If they balance, the odd one is amongst 3, 4, 8. Weigh 3 against 4 to locate odd one.

If they do not balance, note which side is heavier and compare with (C). Depending on this result we either weigh 5 against 2, or 6 against 7.

B3/S



It is quite true that anyone travelling across the Date Line would have to gain or lose a day. However, Puck travelled sufficiently fast to overtake the Sun and the time. When he passed the twelve o'clock midnight time he would also have to pass from one day to another. This latter change will always oppose the change made at the Date Line, causing Puck to arrive back the day he set out.

There is no actual time at the North Pole, only a rate of time. (For the benefit of Polar bears, Laplanders, explorers, etc., G.M.T. is used).

B4/S

Take one flywheel made by each machine and find their total weight. Compare this with the weight of the equivalent number of good flywheels to obtain the discrepancy (over- or underweight).

Now take 1 article from machine No. 1
2 articles from machine No. 2
etc.
and weigh these against the correct weight for that number of parts.

B5 /S

Today's date must be 1st January, 1959.
My birthday was on 31st December, 1958.

B6 /S

Let H be the coins that are suspected of being heavy, and L those that are suspected of being light. We must consider two cases. First, the case where something is already known about the coins, i.e., that a number must have a heavy one among them or others a light one (this may be known as the result of a weighing). Secondly, the case where nothing at all is known about the coins, i.e., before the first weighing, or as a result of a weighing in which the scales balanced and the rogue coin was among those we did not weigh.

First Case
Number of weighings

2 can detect the faulty one amongst 9 coins
3 „ „ „ „ „ „ 27 „
4 „ „ „ „ „ „ 81 „

For example, suppose we have 27 coins and one of 14 is suspected of being heavy or one of the other 13 is suspected of being light.

Weigh as follows:—

$5H$ and $4L$......$5H$ and $4L$ leaving $5L$ and $4H$

This is weighing number one. With each of the three possible motions of the balances (the left side being overweight, the right side being overweight, or the two sides balancing perfectly), we are left with 9 coins and two weighings.

Weigh as follows:—

$2H$ and $1L$......$1L$ and $2H$ leaving $2L$ and $1H$

This is the second weighing and, whatever the motion of the balance, we are left with three coins and one weighing. The two similarly suspected coins are then weighed against each other and the outcome will isolate the faulty coin.

SECOND CASE

Number of weighings

2	can	detect	the	faulty	one	amongst	3 coins
2	,,	,,	,,	,,	,,	,,	$4+1P$ coins
3	,,	,,	,,	,,	,,	,,	12 coins
3	,,	,,	,,	,,	,,	,,	$13+1\,P$ coins

P is a coin that is known to be perfect. An example of the faulty coin amongst 12 in three weighings has been written out in full as a solution to problem number **B**/2.

The general solution to this problem runs as follows:—

		120	
(1)..............	40	40	40
(2)............	$13+P$	14	13
(3)............	$4+P$	5	4
(4)............	$1+P$	2	1

And the last weighing to determine the actual coin. If it is necessary only to isolate the coin without finding out whether it is lighter or heavier, 121 coins may be tackled with five weighings.

B7/S

The minimum number of weights required is five, and these should weigh 1, 3, 9, 27 and 81 pounds. If this solution is not clear to the reader, reference should be made to solution number **B6/S**.

B8 /S

The curate knew his own age and therefore he should have been able to work out from the factors of 2450 the ages of the parishioners. He failed, because there are two sets of factors which give a correct solution.

i.e., $7 \times 7 \times 50$
and $10 \times 5 \times 49$.

$10 \times 5 = 50$ which makes the vicar older than any of his parishioners.

B9 /S

Consider the year 1936. 36 has the following factors:—

1 and 36
2 and 18 i.e., 18th of February
3 and 12 i.e., 3rd December and 12th March
4 and 9 i.e., 4th September and 9th April
6 and 6 i.e., 6th of June.

giving six occasions.

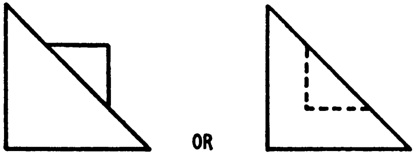
Investigation shows that the years 36, 48, 60, and 72 each give six occasions. Year 24 gives seven occasions!

Areas and Shapes

Solutions____C/S

C1/S

The third projection looks thus:—



C2/S

It is not possible with fewer than six cuts, as can easily be shown thus: consider the centre cube inside the other twenty-six. It has six faces, all of which must be cut.

C3/S

When a knot has four strings passing to it there need not be a loose end in the knot.

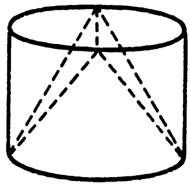
If a knot has three strings entering it there must be at least one end in the knot.

On the parcel there are eight knots with three strings to them. Therefore there must be at least eight ends and hence at least four pieces of string.

C4/S

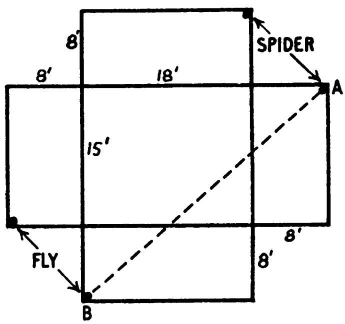
First make from the block a cylinder of diameter 2 in. and length 2 in.

Now cut along the dotted lines to form a wedge-shaped piece. This is the required shape.



C5/S

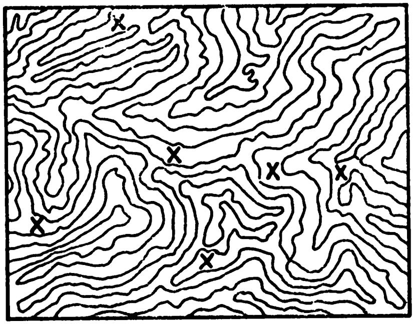
If the walls and the ceiling of the room could be taken and laid out flat they would look something like this:—



From the diagram the spider should take the path A—B which is

$$\sqrt{23^2+26^2}$$
$$=33\cdot75 \text{ ft.}$$

C6/S



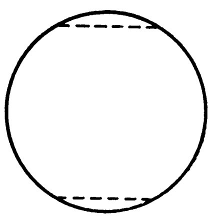
All these points must lie on the same side of the string because the string must be crossed an even number of times to get from one cross to the next.

C7/S

Each man walks directly towards the man on his right whose motion is always at right angles to the man who is walking towards him. Each man therefore walks the length of one side of the field in order to reach the centre.

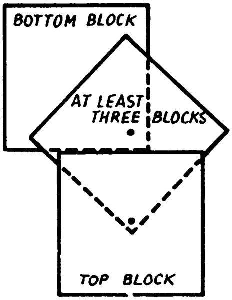
C8/S

The third elevation will look thus:—



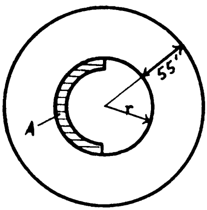
C9/S

This problem may be solved with a minimum of five bricks (or cubes) thus:—



Taking moments about the edge of the bottom block will show that at least three blocks are required to counterbalance the top cube. The centre of gravity must stay inside the bottom block.

C10/S



Let nomenclature be as shown.

$$\text{The total area of the field} = \pi(r+55)^2$$

$$\text{The central area} = \pi r^2 - A$$

$$\text{Where } 2A = \pi r^2 - \pi(r-5)^2$$

$$= 10\pi r - 25\pi$$

But twice the central area = total area.

$$\text{i.e. } 2\pi r^2 - \pi r^2 + \pi(r-5)^2 = \pi(r+55)^2$$

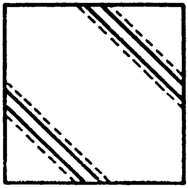
$$\text{i.e. } r^2 - 120r - 3000 = 0$$

Giving $r = 141$ ft.

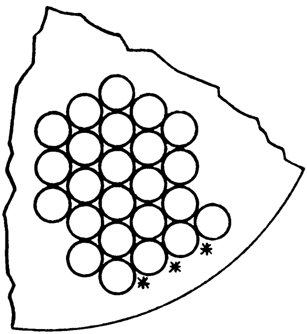
Therefore total area of the field = 13,300 sq. yd.

C11/S

The dove-tails run obliquely, and parallel, thus:—



C12/S



We can build concentric hexagons containing 1, 6, 12, 18, 24, 30, 36, and 42 circles.

When R/r becomes sufficiently large there will be room for extra circles as indicated by * above.

If there is an even number of circles per side in the last hexagon, an 'outsider' can be placed centrally if

$$R/r \geqslant \frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}} \quad \text{i.e. if } R/r \geqslant 13{\cdot}9$$

Two more 'outsiders' can be put each side of this one if

$$\left[(R+r)^2\left(\frac{\sqrt{3}}{2}\right)^2+(2r)^2\right]+r\leqslant R$$

$$\text{i.e. if } 0\leqslant\frac{R^2}{r^2}-14\frac{R}{r}-15$$

$$\text{i.e. if } 0\leqslant\left(\frac{R}{r}+1\right)\left(\frac{R}{r}-1\right)$$

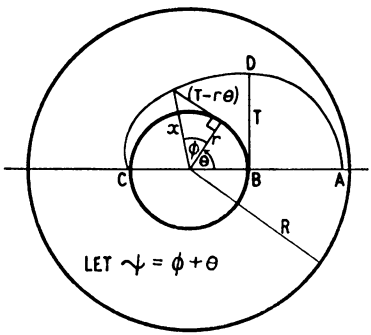
$$\text{i.e. if } R/r\geqslant 15$$

Hence in the example given three 'outsiders' can be accommodated.

The number of saucers that can be placed on the table is:—

$$1+6+12+18+24+30+36+42+(3\times 6)=187$$

C13/S



We know that $T=\pi r$

Let us assume that $R\geqslant T+r$

$$\text{Area BCD}=\tfrac{1}{2}\int_{\phi_1}^{\pi} x^2 d\psi+\tfrac{1}{2}rT \text{ where } \tan\phi_1=\frac{T}{r}$$

$$=\tfrac{1}{2}\int_{\phi_1}^{\pi}[r^2+(T-r\theta)^2][d\phi+d\theta]+\tfrac{1}{2}rT$$

But $\quad r\tan\phi \;=T-r\theta$

$\therefore \sec^2\phi\; d\phi=-d\theta$

Hence Area BCD $=\frac{1}{2}\int_{\phi_1}^{0} r^2 \sec^2\phi\,(-\tan^2\phi)\,d\phi+\frac{1}{2}rT$

$$=\frac{r^2}{2}\left[\frac{1}{3}\tan^3\phi\right]_0^{\phi_1}+rT$$

$$=\frac{r^2}{6}\pi^3+\frac{1}{2}rT$$

Now the goat can eat twice the area ABCD less the area of the mausoleum;

i.e. it can eat $\left\{\frac{r^2}{3}\pi^3+rT+\frac{\pi}{2}T^2-\pi r^2\right\}$, which is half the area of the field.

Hence, $\frac{\pi}{2}(R^2-r^2)=\frac{r^2}{3}\pi^3+\frac{\pi}{2}T^2$

$$\text{giving }\left(\frac{R}{r}\right)^2=\frac{5}{3}\pi^2+1$$

The area of the field $=\pi r^2\left(\frac{R^2}{r^2}-1\right)$

$$=\frac{5}{3}\pi^3\,r^2\text{ sq. yd.}$$

giving an area of 9 acres (approximately).

C14/S

A to F is 65 miles because $65^2=63^2+16^2$
$=56^2+33^2$
$=60^2+25^2$
$=52^2+39^2$

No smaller square can be decomposed into four pairs of squares. This can be proved by trial, but the argument can be shortened. We note that 3, 4, 5 and 5, 12, 13 are the two smallest integral solutions of a right-angled triangle (6, 8, 10 is a duplication of 3, 4, 5). Thus 5×13 has the two solutions 5 (5, 12, 13) and 13 (3, 4, 5). Moreover, 65 is the sum of 8^2+1^2 and $7^2=4^2$, and hence two more solutions arise.

Humorous

Solutions________________ ________________D/S

D1 /S

Casual readers please note: the question was not 'How fast can the bicycle go?' but 'How did Ernest find out the speed?'

First of all he turned the bicycle upside down and fastened the hairpin to the rear wheel fork. He then turned the rear wheel until the hairpin, drumming on the spokes of the wheel, sounded the same as the *A* on James' clarinet. By knowing where *A* stood on the frequency scale and counting the spokes on the wheel Ernest could calculate the wheel speed for a given pedal speed. He would then know the speed of the bicycle.

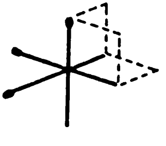
D2 /S

The train travelling against the spin of the earth will wear its wheels out more quickly, since the centrifugal force is less on this train.

D3 /S

None: the bus-driver happened to be walking home after a days work!

D4 /S



D5 /S

We don't recommend that you try this one out! The mechanic took one of the gauges in each hand and rubbed them together until the 4 thou. feeler gauge had been completely worn away. The 14 thou. gauge must then have been 10 thou. thick.

D6 /S

All London Transport buses are diesel driven and use oil, not petrol.

D7 /S

The bear must have been white; the assumption may be that the man could only have started out from the North Pole—but this is not necessarily true.

A man setting out towards the South Pole from a point 11½ miles from this pole could stop after walking 10 miles and walk 10 miles East. This would take him right round the South Pole and on to the same line of Longitude that he walked down. He could then retrace his steps by walking 10 miles North. It is just as well that there are no brown bears at the South Pole or you would have the answer only partly right.

D8 /S

If you want an afternoon of hilarious entertainment you should try this problem for yourself! In practice old, loose-fitting clothes are best because it is necessary to feed large portions of each garment through the loop of the handcuff. Shoes, socks, collar, tie, etc., present no difficulty and the method of removing trousers should be sufficient explanation for the remainder.

The trousers should be loosened around the waist and the whole of one leg of material fed through the legcuff. Once the foot is free of the trouser, the empty trouser leg must be fed back through the legcuff. The complete pair of trousers may then be slipped through the other legcuff and away. Repeat this procedure for jacket, shirt, and underwear!

"Cross-numbers"

Solutions________________________________E/S

E1/S

$a = 3$
$b = 12$
$c = 4$

1	6	7	6
2	4	4	9
9	6	1	0
6	9	1	0

E2/S

1 2	2 9	3 0	0	2	4 0	5 9
6 7	5	4	7 8	6	4	2
8 7	0	5	3	9 5	0	7
10 2	6	7	4	8	2	1
11 1	2	2	5	12 5	6	13 2
14 6	4	3	6	3	4	3

E3 /S

1	3	3	4	0	2
4	2	8	5	7	1
6	7	6	2	0	1
4	6	9	1	8	2
1	8	1	3	3	5

E4 /S

9	8	0	1	6	6	7	2
3	1	4	2	4	1	5	9
2	2	1	7	0	8	3	4
1	4	2	7	7	6	1	2
1	2	3	4	5	6	7	9
1	0	0	1	3	6	2	4

E5 /S

6	3	2	5	5	8
5	9	6	2	6	1
2	2	9	0	2	6
1	6	9	3	1	4

$a=7,\ b=11,\ c=5,\ d=9$

1	2	1	1	1
5	3	0	2	1
1	1	8	5	8
1	1	9	8	8
4	9	3	4	3

Analytical

Solutions____F/S

Let the present age of the monkey be x years and the present age of the monkey's mother be y, and the present age of the brother be z.

The last paragraph tells us that $x+y=2z$ and from the first paragraph $x+y=4$. Also from the first paragraph $W+\frac{l}{4}=\frac{3y}{2}$ where l is the length of the rope. Paragraph three can be written as:—The monkey's mother $(y-x)$ was twice as old as the monkey $(x-X)$ was when the monkey's mother $(y-X)$ was half as old as the monkey $(x-Y)$ will be when the monkey $(x-Y)$ is three times as old as the monkey's mother $(y-Z)$ was when the monkey's mother $(y-Z)$ was three times as old as the monkey's present age (x).

Hence:—

$$\begin{aligned}(y-X)&=2(x-X)\\(y-X)&=\tfrac{1}{2}(x-Y)\\(x-Y)&=3(y-Z)\\(y-Z)&=3x.\end{aligned}$$

Eliminating X, Y, and Z we find that

$$4y=13x.$$

We now have two equations in x and y giving $x=16/17$ years and $y=52/17$.

Likewise from paragraph two we obtain:—

$$\text{Weight }(W)=\frac{l}{4}+11x-2y.$$

We now know the value of x and y and we have two equations in W and l. This gives l to be $12/17$ feet.

F2/S

Let A people drink beer and a people not.
Let B „ „ wine „ b „ „
Let C „ „ water „ c „ „

From cost $2.A+5.B+C=293$
Used glasses $A+B+C=106$

therefore $A+4B=187$.

But we are told $B=39$, so $A=31$ and $C=36$.

If N is the number of guests then $N=a+A$
$=b+B$
$=c+C$

and as c is given as 18 we can find that N is 54 and $a=23$
$b=15$

We also know that teetotallers $=9$ i.e. $(abc)+(abC)=9$ (where (abC) is number of people who drank no beer and no wine)

Consider:—

We are told (AbC) must be max. ∴ $(AbC)=6$
and $(Abc)=0$
also $(aBc)=3$ from the question

Now $(abc)+(aBc)+(ABc)+(Abc)=18$
i.e. $(abc)+(3)+(ABc)+(0)=18$
i.e. $(abc)+(ABc)=15$

But (abc) must have a value between 0 and 9 therefore (ABc) must lie between 6 and 15. But $(ABc)+(ABC)=25$

Therefore (ABC) lies between 10 and 19

Therefore (ABC) must be at least 10.

F3/S

A strip with one twist, when cut, will give a loop with two twists (not a single surface any longer).

A strip originally with two twists, when cut, will give two loops linked together like a paper chain, each with two twists.

F4/S

Let the present age of the ship be x years and of the boiler be y years.

Hence:—

A ship (x) is twice as old as its boiler ($y-X$) was when the ship was ($x-X$) as old as the boiler is now.

$\therefore x=2(y-X)$

and $(x-X)=y$

Eliminating X gives $4y=3x$

Also, $x+y=30$

Therefore y (the boiler) $=\frac{90}{7}$ years, and x (the ship) $=\frac{120}{7}$ years.

F5/S

	G (M)	LB (B)	RB	LH (M)	CH Capt (M)	RH (B)	OL	IL (M)	CF (M)	IR (B)	OR (B)
PARKS (B)	g	n	/	k	j	*	/	l	g	a	a
GARDINER	/	f	f	/	*	/	k	k	k	k	k
RAKES (V.C.)	/	/	/	*	k	/	k	k	k	k	k
DALE (B)	g	f	f	h	j	l	*	l	g	a	a
SWIFT (M)	*	l	c	h	h	h	c	c	c	c	c
JAMES (B)	g	n	*	h	j	n	/	l	g	a	a
EVANS (M)	/	m	/	/	o	m	/	*	p	m	m
SMITH (M)	/	m	/	b	b	b	/	/	*	m	m
BURNS	/	*	/	d	o	/	/	o	o	o	o
ROBINSON	/	/	/	/	o	/	/	/	/	*	p
JONES	/	f	f	/	o	/	/	/	/	/	*

The letters refer to the statements about the players. M—Married. B—Bachelor.

* indicate players' true positions.

F6/S

The first man, whose reply was lost, could only have said one thing when asked what his colour was because if he had worn black he would have had to lie.

Therefore the 1st man was in white the 2nd man was in black and the 3rd man was in white.

F7/S

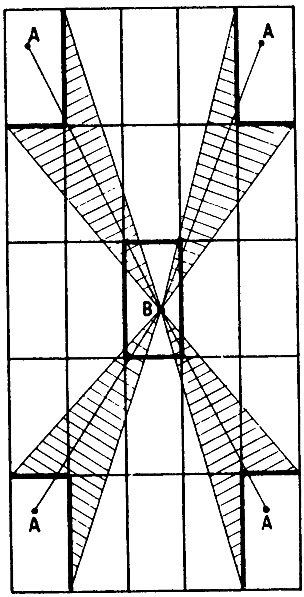
At one end of the cable connect the wires into bunches of 1, 2, 3, 4, etc., and label these bunches A. B. C. D. etc. In a cable of, say, fifty wires there will be nine bunches with five left over, which can be made up into the tenth bunch.

Walking to the other end of the cable and checking the 'ends' with battery and bulb will identify all but two groups. The wires at this end should be individually numbered and assembled into different groups, i.e., one from each of the original groups except B, one from each group except C, etc. One wire can be left unconnected in any group and the previous lone wire can be used to distinguish between the two identical groups. Returning to the first end of the cable, all the wires may be tagged, requiring one more journey up and down the cable to disconnect all the wires. A total of four journeys is required.

Cards and Games

Solutions ________________________________ G/S

G1/S



From the image diagram one can see that the ball at A can be hit after a suitable number of rebounds by cueing the ball along one of four narrow arcs. It has been assumed that the table is twice as long as it is wide, which in practice is very nearly true.

Maximum width of arc possible $= \tan^{-1} \frac{3}{2} - \tan^{-1} \frac{1}{2}$

$$= 29^\circ\, 45'$$

Minimum width of arc possible $= \tan^{-1} \frac{1}{2} - \tan^{-1} \frac{1}{6}$
$= 17° 9'$

It may easily be calculated that:—

(*a*) It is pointless aiming within 18° 24′ of the vertical.
(*b*) It is pointless aiming within 33° 42′ of the horizontal.

G2 /S

The second player should always win since he enters the even sides and hence the fourth side.

He must never complete a third side unless doing so is unavoidable, when he should give away as few squares as possible.

G3 /S

Played	*Won*	*Drawn*
21	15	4
21	14	4
20	12	6
20	13	3
18	10	7
22	8	4
22	7	6
21	7	4
21	7	3
19	6	1
21	3	6
21	2	5
20	1	5
19	0	3

8 games in a match.

G4 /S

6S – 7H /AD out /5H – 6S /4S – 5H /QC up /JH – QC /10C –JH /9D – 10C /8S – 9D /AC out /5C up /5D – 6C /5C – 6D /4H –5C /2 and 3D out /4S to 7H – 8S /2C out /JD – QS /JS and 10D

up /4C – 5D /QH – KC /JS and 10D – QH /8C up /KS up /AH out /AS out /4D out /2H out /3H out /4H out /8H up /3C out /4C out /5C out /5D out /6D out /6C out /7C out /8C out /9S – 10D /8H – 9S /7S – 8H /JC up /9C out /5S up /KD up /2S out /3S out /4S out /5S out /7D out /5H out /6S out /7S out /JD and QS – KD /9H up /6H out /7H, 8H, 9H out /10H out /8D out / and rest is easy.

G5 /S

6H – 7S /QH – KC /JC – QH /5C – 6H /4H – 5C /6S – 7H / 5D – 6S /AD out /2D out /3C – 4H /8H – 9S /7C – 8H /9C up /8D – 9C /AS out /AH out /10C – JD /9H – 10C /8S – 9H /7H to 5D – 8S /4C – 5H /9D up /3D out /5D to 8S – 9D /JD to 9H up /4D out /5D out /8S to 6S – 9H /5H and 4C – 6S /3C to 6H – 7C /7S – 8D /10H – JC /3S up /8C – 9D /2H out /2S out /3S out /4S out /9C to 7S – 10D /KC to 10H up /7D – 8C /JS up /AC out /3H out /5S out /10D to 7S – JS / and rest is easy.

G6 /S

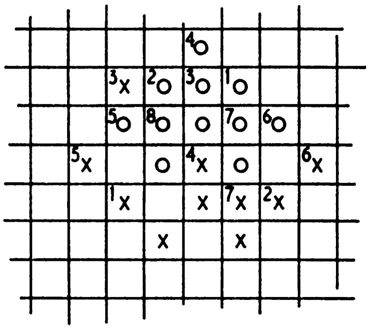
9C – 10D /8H – 9C /7S – 8H /6H – 7S /JC – QD /10S – JH /9D – 10S /3D – 4S /8S – 9D /7H – 8S /AC out / 6S – 7H /QH – KC /7C up /QC – KH /2C out /5H – 6S /3C out /4C out /5C out /4S and 3D – 5H /9S up /6C out /7C out /KS up /AH out /KH and QC up /10H – JC /9H – 10C /8C – 9H /9S – 10H /KC and QH up /AS out /5S – 6H /JC – QH /3H up /2H out /3H out /10H and 9S – JS /QD and JC – KS /4D – 5S /5D to 10D – JC /8C up /2 and 3S up /3D to JH – QC /7D up /6D up /8D – 9S /JD up /AD out / and rest is easy.

G7 /S

QH – KS /JC – QH /10D – JC /3C – 4H /5C up /AC out /2C out /3C out /4C out /5C out /9C – 10D /JH and 10C – QC /9S – 10H /10S up /AS out /4S up /9H – 10C /8C – 9H /AD out /3D – 4S /2S out /7D – 8C /6S – 7D /5D – 6S /4S and 3D – 5D /KC up /3S out /8S and 9D – 10S /8S, 9D and 10S – JD /9C to QH – KC /3D to 7D – 8S /KS up /QD – KS /JS – QD /9S and 10H – JS /4H –

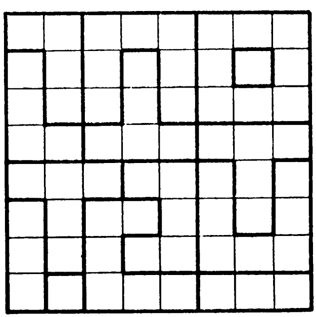
5S /2D out /6C out /7C out /3D out /4D out /4S out /5D out /8C out /9C out /9H, 10C, JH up /QC and KD up /9H up /10C out /JH – QC /10C borrowed – JH /9H – 10C /AH out /6D out /2H out /4H up /5S out /3H out /4H out /5H out /8H – 9S /8D up /6H out /7H out /8H out / and rest is easy.

G8 /S



Nought must complete three in a row at every stage in order to dominate the play.

G9 /S



The discussion centres round the number of shuffles necessary to bring a pack back to its original order. I find it more convenient to consider a type of shuffling which is the reverse of Aunt Agatha's. We take a pack (say on the right) and transfer it to the left, by taking cards one at a time from the top and bottom alternately, starting from the bottom, and putting them one on top of the other on the left.

Suppose Aunt Agatha had a pack of five on the left numbered 1, 2, 3, 4, 5 from the bottom. After one shuffling, her order from the bottom would be—1, 3, 5, 4, 2. Another shuffle would give 1, 5, 2, 4, 3. Successive shufflings thus give:—

$$\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 4 & 2 \\ 1 & 5 & 2 & 4 & 3 \\ 1 & 2 & 3 & 4 & 5 \end{array} \quad \ldots\ldots\ldots\ldots(a)$$

returning to the original order in three shuffles.

With the reversed type of shuffling we should get:—

$$\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 2 & 4 & 3 \\ 1 & 3 & 5 & 4 & 2 \\ 1 & 2 & 3 & 4 & 5 \end{array} \quad \ldots\ldots\ldots\ldots(b)$$

(*b*) is the reverse of (*a*) and it is easily seen that this is true for a pack of any size. From here onwards I consider the second type of shuffling.

Consider a pack of n. It is clear that, in successive shufflings, the card numbered 1 always stays at the bottom, i.e. in the first place. The card numbered 2 goes to third place on the second shuffle, the fifth place on the third shuffle, and generally to the $(2^{r-1}+1)$th place on the rth shuffle, provided that $(2^{r-1}+1) \leqslant n$. Thus the card 2 travels through the pack until it can go no further to the right (in the order as written out above). If it is then in the mth place, it will go to the $[2(n-m)+2]$th place on the next shuffle; and so on.

Now the second card cannot occupy more than $(n-1)$ places altogether. Hence if the shuffling continues indefinitely, the second card must return to one of the places it has previously occupied, after which it retraces its steps. I give the name cycle to the series of places it occupies before returning to the starting point. Thus in the example given above (*b*), the cycle is 2, 3, 5 the numbers referring to the ordinals of the places it occupies. Clearly, since

the second card begins in the second place, 2 will form a member of the cycle and the second card must always return to place 2.

Further, each card which starts in the place corresponding to a number in the 2-cycle will go through the same cycle: e.g. in the above example 3 goes through the cycle 3, 5, 2 and 5 through the cycle 5, 2, 3.

Thus, when 2 has returned to its starting point, all other members in its cycle will also have returned to their starting points.

The same will be true of any other card. In a single shuffle the mth card will be moved to place $(2m-1)$ if $m \leqslant \frac{n}{2}$ and to place $[2(n-m)+2]$ if $m > \frac{n}{2}$. Each card will describe a cycle and the n

ORDINALS

NUMBER OF CARDS IN PACK	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
40	–	39	1	10	2	35	11	23	3	20	36	17	12	28	24	6	4	15	21	8	37	33	18	26	13
41	–	27	1	$\boxed{9}$	2	23	$\boxed{1}$	16	3	III	24	13	$\boxed{2}$	8	17	$\boxed{5}$	4	6	I	21	25	$\boxed{7}$	14	11	$\boxed{3}$
42	–	41	1	31	2	27	32	8	3	21	28	24	33	36	9	17	4	15	22	6	29	39	25	19	34
43	–	8	1	$\boxed{8}$	2	IV	$\boxed{1}$	H	3	h	I	$\boxed{5}$	$\boxed{2}$	c	A	$\boxed{D}$	4	$\boxed{b}$	a	f	II	6	$\boxed{6}$	F	$\boxed{3}$
44	–	28	1	$\boxed{14}$	2	8	$\boxed{1}$	12	3	$\boxed{5}$	9	25	$\boxed{2}$	18	13	$\boxed{8}$	4	21	$\boxed{6}$	23	10	$\boxed{12}$	26	6	$\boxed{3}$
45	–	11	1	$\boxed{11}$	2	XI	$\boxed{1}$	$\boxed{5}$	3	IV	I	8	$\boxed{2}$	K	$\boxed{6}$	C	4	VII	V	I	II	IX	9	$\boxed{9}$	$\boxed{3}$
46	–	12	1	$\boxed{12}$	2	$\boxed{5}$	$\boxed{1}$	VI	3	L	$\boxed{6}$	$\boxed{9}$	$\boxed{2}$	c	I	E	4	8	A	C	$\boxed{7}$	IV	$\boxed{10}$	10	$\boxed{3}$
47	–	10	1	$\boxed{5}$	2	X	$\boxed{1}$	J	3	e	I	VII	$\boxed{2}$	IV	A	$\boxed{E}$	4	F	a	D	II	c	VIII	8	$\boxed{3}$
48	–	36	1	31	2	$\boxed{9}$	32	15	3	26	$\boxed{1}$	12	33	23	16	$\boxed{4}$	4	19	27	II	$\boxed{2}$	10	13	29	34
49	–	24	1	19	2	$\boxed{24}$	20	$\boxed{15}$	3	14	$\boxed{1}$	11	21	$\boxed{12}$	$\boxed{16}$	$\boxed{19}$	4	$\boxed{4}$	15	$\boxed{8}$	$\boxed{2}$	$\boxed{10}$	12	$\boxed{22}$	22
50	–	15	1	$\boxed{5}$	2	XV	$\boxed{1}$	VIII	3	E	I	c	$\boxed{2}$	XII	IX	$\boxed{E}$	4	9	A	IV	II	$\boxed{C}$	a	VI	$\boxed{3}$
51	–	50	1	19	2	24	20	9	3	38	25	13	21	16	10	43	4	30	39	46	26	28	14	36	22
52	–	51	1	9	2	29	10	14	3	18	30	35	11	48	15	38	4	41	19	25	31	23	36	33	12

cards may be formed into a number of groups, each of which constitutes a cycle.

Thus in the example given, 4 always remains in the same place and thus has a cycle of 1. 2, 3 and 5 form a cycle of 3.

Now let there be cycles $k_1, k_2, \ldots k_p$ for n cards and let N be the least common measure of $k_1 \ldots k_p$. Then in N shuffles, and not before, all the cards will return to their original places. (As a matter of fact it may be shown that the cycle of the card 2 is a multiple of all other cycles, so that N is in fact the number of the 2 cycle. This result, however, is not strictly necessary for the solution of the present problem, although it contributes to an easier solution).

The problem is then to find the size of the cycles of the different

ORDINALS

26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
31	29	30	25	32	7	14	5	27	16	19	22	34	9	38												
19	9	–	18	10	$\boxed{6}$	20	5	$\boxed{4}$	7	12	II	15	22	$\boxed{8}$	26											
13	37	11	10	12	18	38	5	14	16	35	23	20	7	26	30	40										
$\boxed{B}$	d	D	B	C	$\boxed{A}$	E	5	e	$\boxed{a}$	$\boxed{C}$	b	$\boxed{4}$	g	G	III	$\boxed{7}$	7									
16	19	$\boxed{10}$	14	–	$\boxed{9}$	15	5	$\boxed{11}$	22	20	$\boxed{7}$	17	24	$\boxed{4}$	11	7	$\boxed{13}$	27								
6	A	G	$\boxed{7}$	E	D	F	5	$\boxed{8}$	VIII	H	VI	B	J	7	III	$\boxed{4}$	X	$\boxed{10}$	10							
J	a	6	II	H	F	G	5	I	9	III	B	7	D	b	$\boxed{8}$	K	V	$\boxed{4}$	$\boxed{11}$	11						
H	V	$\boxed{C}$	B	6	$\boxed{A}$	–	5	$\boxed{B}$	G	7	b	C	E	$\boxed{D}$	III	VI	d	I	IX	$\boxed{4}$	9					
$\boxed{7}$	24	21	17	8	$\boxed{5}$	6	5	7	20	$\boxed{6}$	28	9	I	18	$\boxed{3}$	22	11	25	14	$\boxed{8}$	30	35				
17	$\boxed{13}$	9	$\boxed{17}$	$\boxed{6}$	$\boxed{20}$	7	5	6	$\boxed{5}$	8	16	$\boxed{21}$	$\boxed{9}$	$\boxed{7}$	$\boxed{3}$	18	$\boxed{11}$	10	13	$\boxed{14}$	$\boxed{23}$	18	23			
13	XIII	C	X	7	$\boxed{A}$	11	5	–	10	6	B	12	V	$\boxed{B}$	III	8	$\boxed{D}$	XI	b	D	VII	XIV	$\boxed{4}$	14		
48	17	7	11	41	44	34	5	32	31	33	40	6	47	35	27	45	29	42	15	12	37	8	23	18	49	
7	49	27	16	46	39	21	5	44	42	43	20	45	26	6	32	22	24	40	37	47	34	17	13	28	8	50

cards. The method may be illustrated by a simple example for $n=11$. We write the numbers 1 to 11, which are to be considered as the ordinals of the cards in the pack.

1	2	3	4	5	6	7	8	9	10	11
–	6	1	$\boxed{3}$	2	4	$\boxed{1}$	–	3	$\boxed{2}$	5

The card 2 goes on first shuffling to place $2(2)-1=3$; we therefore put a 1 under the 3. The second shuffling takes it to $2(3)-1=5$; i.e. the fifth place. The third shuffling to $2(5)-1=9$th place. We therefore write 2 and 3 under the 5 and 9 respectively. We are now more than half way across the pack, so that the fourth shuffling will bring card 2 to the $2(11-9)+2=6$th place. We write 4 under the 6. The fifth shuffling leads to place 11 and the sixth back to 2; hence we write 5 and 6 under 11 and 2. This completes the cycle, which thus includes the 2nd, 3rd, 5th, 6th, 9th and 11th cards. The next card, not included in the cycle, is the fourth and we find that this runs through the cycle 4, 7, 10 shown by the numbers in squares above. The remaining digit, 8, remains fixed and thus has a cycle of 1.

The cycles are thus 6, 3, 1 and after six shuffles the pack of 11 returns to its original arrangement.

The attached table on pages 132 and 133 shows the cycles for $n=40$ to 52.

To solve Aunt Agatha's problem, only the cycle containing the 2 card is really needed.

Five cards were missing.

G11/S

The solution is similar to that of the previous problem; the cards moving in cycles. Beginning with the 2-card we find the cycle by doubling and subtracting one until over half-way through the pack. Then we double the difference between the ordinal number and half the total number of the pack, e.g. for the 2-cycle with 52 cards.

$$2,\ (2\cdot2-1)=3,\quad (2\cdot3-1)=5,\quad (2\cdot5-1)=9,\quad (2\cdot9-1)=17$$
$$(2\cdot17-1)=33,\ 2(33-26)=14,\ (2\cdot14-1)=27,\ 2(27-26)=2$$

i.e. eight shufflings.

The number of shufflings required for packs of 2 to 52 are shown.

It is required to find n such that the number of shuffles required for $(52-n)$ cards is n. The only solution is 40 cards requiring 12 shuffles. Hence Uncle George took 12 cards.

Number of cards in pack	*Number of shuffles required*
2	1
4	2
6	4
8	3
10	6
12	10
14	12
16	4
18	8
20	18
22	6
24	11
26	20
28	18
30	28
32	5
34	10
36	12
38	36
40	12
42	20
44	14
46	12
48	23
50	21
52	8

Chess

Solutions H/S

H1 /S

Black won by promoting a Pawn to a Bishop after P(Kt 7) Kt 8.

He could not have promoted his Pawn on his K R 8 because this would involve 15 captures and White has lost only fourteen pieces.

H2 /S

White must allow Black to take the front three Pawns. He then advances the fourth and uses the fifth to waste a move at the right moment without stalemating.

H3 /S

Black mates in one. If it were White's move he could mate by P—K Kt 5. Black can mate by Kt × P double check. But it can be proved that Black cannot have moved last.

It will be clear that Black cannot have moved either Rook, the Bishop, the Queen, or the King. Black's Queen's Rook cannot have got out and one of his Rooks must be a promoted Pawn. This must have been promoted on his Q B 8 involving two captures. Likewise White's Q B must have been captured *in situ*, not by a Pawn. It will be found that all captures are necessary to account for the existing situation and therefore the last move by Black cannot have been Kt × piece on White K R 2. Similarly no other Pawn captures by Black could have taken place on the last move. Therefore White moved last.

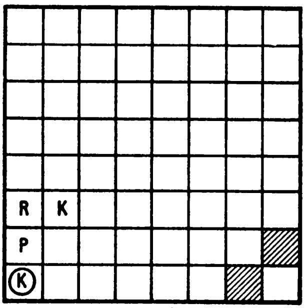
If Black's last move were P (K Kt 4) ×piece on K B 5 or P (K R 5) ×piece on K B 6, this would involve four Pawn captures whereas White can only have lost three by Pawn capture.

H4 /S

White's reply should have been 'It's not your move!'—Both had had an odd number of moves.

H5 /S

BLACK



WHITE

The board could not have been the other way up because Black cannot have placed himself in such a position unless White's pawn threatened the other way. White mated by moving his Rook to Q R 4, then to Q B 4 and to mate at Q 1.

H6 /S

Black's Bishop must be a 'Queened' Pawn since he could not have moved the King's Bishop from its original square. However, White's King or Rook must have been moved to allow a Pawn through. Therefore as a penalty White must move his King. He can only move to the left and Black mates with his Queen on K B 8.

H7 /S

It must have been on Q 2.

White mates by Pawn taking Pawn *en passant*, since Black's last move must have been Pawn to Bishop 4. This can be demonstrated from the position.

H8 /S

The chess board has been turned round so that Black must have been playing from the left hand edge of the board. Black castled to mate.

Probability

Solutions—J/S

J1 /S

Suppose there are n letters, and consider an unterminating random series of the letters; the chance that any particular letter will be a specified letter is $1/n$. The question is, what is the average length of the run in such a series for every letter to appear at least once?

In such a series the expected run required to reach one of $P(Sn)$ specified letters is n/P, say $1/p$.

For the chance that the first is one of the P is p. The chance that the second is one of the P but the first is not, is $(1-p)p$. The chance that the third is one of the P but the first two are not is $(1-p)^2p$ and so on. Thus the average length of run is:

$$p+2\ (1-p)\ p+3\ (1-p)^2p+\ \ldots$$
$$=p[1+2\ (1-p)+3\ (1-p)^2+\ \ldots]$$
$$=\frac{p}{[1-(1-p)]^2}=\frac{1}{p},\text{ as stated.}$$

Hence, if at any stage there are P letters left to be obtained the average length of the run required to get one of them is n/P. Moreover, the runs are independent. Hence the length of run required to get all the letters is

$$\frac{n}{n}+\frac{n}{n-1}+\frac{n}{n-2}+\ \ldots\ \frac{n}{1}$$

$$=n\ (1+\tfrac{1}{2}+\tfrac{1}{3}+\ \ldots\ +\frac{1}{n})$$

By adding the reciprocals we find that, for $n=19$, run$=67\cdot4$; for $n=20$, run$=71\cdot95$ (corresponding to age $70\cdot95$).

Hence there are 20 letters.

J2/S

The problem relates to *losing the lead* which includes ties. If A polls a votes, B polls b votes, $a > b$.

A can lose the lead in two ways:

(i) B gets the first vote and so A loses the lead at once.

(ii) A gets the first vote but loses the lead at some later stage. The number of votes, up to the *first* point when he loses the lead, must be even and must end with a B.

The point is that the number of cases under (i) is the same as the number under (ii); for to any sequence $A \ldots B$ there corresponds one and only one sequence with A's and B's interchanged.

The probability that the sequence begins with a B is $\dfrac{b}{a+b}$.

Thus the chance that A loses the lead is $\dfrac{2b}{a+b}$ and the chance that he never loses the lead is $1 - \dfrac{2b}{a+b} = \dfrac{a-b}{a+b}$. If $a=9$, $b=6$, the chance of not losing the lead $= \dfrac{9-6}{9+6} = \dfrac{3}{15} = \dfrac{1}{5}$.

The odds are 4 to 1 against.

J3/S

Let x be the number of people.

The number of ways they can have birthdays is $365x$.

The number of ways they can have different birthdays is $365 \times 364 \ldots (365-x+1)$.

Thus the chance that a number of birthdays are alike is

$$\frac{365 \times 364 \ldots (365-x+1)}{365^x}$$

$$= 1\left(1-\frac{1}{365}\right)\left(1-\frac{2}{365}\right) \ldots \left(1-\frac{x-1}{365}\right) \ldots 1$$

and this is to be less than $\frac{1}{2}$.

We may either work through this question until we arrive at a figure that is less than a half, or we may use an approximate method.

Since x is relatively small compared with 365.

$$\log_e \pi \left(1-\frac{x-1}{365}\right) \leqslant \log_e \tfrac{1}{2}$$

$$\text{i.e.} -\left(\frac{1}{365}+\frac{2}{365}+\ \ldots\ \frac{x-1}{365}\right) \leqslant -\log_e 2$$

$$\text{as } \log_e(1-Z=-Z+Z^2 \text{ etc.}$$

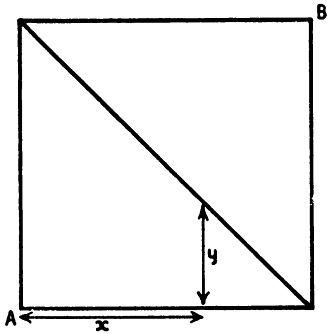
$$\text{i.e. } \frac{\frac{1}{2}x(x-1)}{365} \geqslant \log_e 2 \quad \text{i.e. } x(x-1) \geqslant 730 \times 0{\cdot}6931$$

$$\text{giving } x=23 \text{ approx.}$$

Putting this value in **I** gives $0{\cdot}495$

$\therefore$ Solution is 23 people.

J4/S



Suppose they meet at (x, y). Then A has made x and y steps. B has made $(n-x)$ and $(n-y)$ steps.

$\therefore\ 2n-(x+y)=x+y$

i.e. $x+y=n$

i.e. they must meet on the diagonal shown.

Number of routes for A to (x, y) is $\binom{x+y}{x}=\binom{n}{x}$

Number of routes for B to (x, y) is $\binom{2n-x-y}{n-x}=\binom{n}{n-x}=\binom{n}{x}$

$\therefore$ Number of routes on which they meet $=\displaystyle\sum_{x=0}^{n} \binom{n}{x}^2=\binom{2n}{n}$*

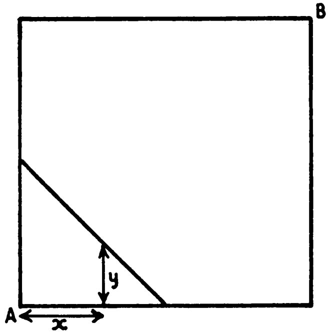
* For $(1+t)^n.\ (1+t)^n=(1+t)^{2n}$. Expand both sides by the binomial theorem and equate coefficients of t^n.

Total number of routes $=\binom{2n}{n}^2$

Probability of meeting $=\dfrac{1}{\binom{2n}{n}}=\dfrac{n!\ n!}{2n!}$

In this case $n=8$.

$\therefore$ Probability $=\dfrac{1}{12870}$



If B goes three times as fast as A, $3(x+y)=n-x+n-y$.

i.e. $x+y=\dfrac{n}{2}=4$ (in this case)

i.e. they must meet on line shown.
Number of routes on which they meet

$$=\binom{4}{0}\binom{12}{4}+\binom{4}{1}\binom{12}{5}+\binom{4}{2}\binom{12}{6}+\binom{4}{3}\binom{12}{7}+\binom{4}{4}\binom{12}{8}$$

But $\binom{4}{0}=\binom{4}{4}$ and $\binom{4}{1}=\binom{4}{3}$ etc.

$\therefore$ Number

$$=2.\frac{12.11.10.9}{1.\ 2.\ 3.\ 4}+8.\frac{12.11.10.9.8}{1.\ 2.\ 3.\ 4.\ 5}+\frac{4.3}{1.2}\cdot\frac{12.11.10.9.8.7}{1.2.3.4.5.6}$$

$= 990+6336+5544$
$= 12870$

Total number of routes $=\binom{2n}{n}^2=(12870)^2$

$\therefore$ Chance of meeting $=\frac{1}{12870}$

J5/S

Given r different things with an equal chance of occurrence, what is the probability that in n trials we shall get one each of the r things?

Consider the favourable cases. The total number r^n contains all these and the cases with $(r-1)$ objects. Subtract these to get $r^n-\binom{r}{1}(r-1)^n$. But this subtracts too much, namely $\binom{r}{2}$ of the cases with $(r-2)$. Add these to get $r^n-\binom{r}{1}(r-1)^n+\binom{r}{2}(r-2)^n$. This adds too many cases with $r-3$ objects and so on.

Hence we get the general formula:—

$$P=\frac{1}{r^n}\left\{r^n-\binom{r}{1}(r-1)^n+\binom{r}{2}(r-2)^n-\binom{r}{3}(r-3)^n\ldots\pm\binom{r}{r-1}(1)^n\right\}$$

The above formula may be established by the difference equation:—

$$u_{n,r+1}=\binom{n}{1}u_{1,r}+\binom{n}{2}u_{2,r}+\ \ldots\ \binom{n}{n}u_{n,r}$$

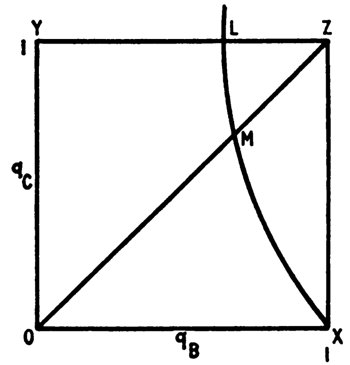
or $\beta^{-1}(1+\alpha^{-1})^n=1$ where $\beta u_r=u_{r+1}$

$\alpha u_n=u_{n+1}$

In this case $r=6$ and by trial when:—

$n=12\quad P=0\cdot438$
$n=13\quad P=0\cdot514$

Hence the landlord had nine dice.



Consider the case where B and C are duelling. Let p_B, p_C be their chances of hitting their man. Let u_B = the probability that B survives if he fires first, and similarly for C.

$$u_B = p_B + (1 - p_B)\ (1 - u_C)$$
$$u_C = p_C + (1 - p_C)\ (1 - u_B)$$

If we write $(1 - p_B) = q_B$ and $(1 - p_C) = q_C$ the above equations give

$$u_B = \frac{1 - q_B}{1 - q_B q_C} \quad u_C = \frac{1 - q_C}{1 - q_B q_C}$$

C fires at A and we are told that there is uncertainty whether it is to C's advantage to hit A or not.

If C hits A, B fires at C and C's chance of survival is $(1 - u_B)$ .... (1)

If C misses A then B fires at A

B hits A	B misses A
C fires at B	A hits B
$p_B\ u_C$	C fires at A
	$q_B\ p_C$ .... (2)

From (1) and (2)

$$1 - u_B = p_B\ u_C + q_B\ p_C$$

substituting for u_B and u_C we get

$$q_C=\frac{1-q_B}{q^2_B} \quad \dots\dots\dots\dots\dots\dots\dots\dots (3)$$

We have to find bounds for q_B such that (3) is true, $q_C>q_B$ and both q_C and q_B lie in the interval (0, 1).

The curve $q_C=\frac{1-q_B}{q^2_B}$ meets OZ in M and YZ in L. The domain of variation must lie in the square $OYZX$ and inside the triangle OYZ (because $q_B<q_C$).

Thus the limits are the piece of curve LM and we have to find q_B at L and M.

q_B at L $\quad (q_C=1)$ i.e. $1=\frac{1-q_B}{q^2_B}$

$$\text{i.e. } q^2_B+q_B-1=0 \quad \therefore \; q_B=\frac{-1+\sqrt{5}}{2}=0{\cdot}618.$$

q_B at M $\quad (q_B=q_C) \quad q^3_B+q_B-1=0$ positive root $0{\cdot}683$.

Thus the limits for q_B are $0{\cdot}618$ and $0{\cdot}683$ and those for p_B are $0{\cdot}317$ and $0{\cdot}382$.

Alternative Solution

Suppose the probability of B's hitting his man is b, and that of C to be c. If A were eliminated and a duel started between B and C in which C got the first shot, the probability p of C's victory is given by $p=c+(1-b)p$. If C hit A and B got the first shot in such a duel, C's chance would be $(1-b)p$.

If C fails to hit A then B *must* try to do so (for A, unhit, would aim at the better shot). The probability that he does so is b, and C's chance is then p. The probability that he fails is $1-b$ and C's chance is c, based on one shot at A.

Hence it is in C's interest *not* to hit A if $bp+(1-b)c>(1-b)p$

$$bp>(1-b)\,(p-c)$$
$$bp>(1-b)^2\,(1-c)p$$
$$c>1-b/(1-b)^2$$

This *must* be true if $b>(1-b)^2$ i.e. $b>0{\cdot}3820$ and *can only* be true if $b>c>1-b/(1-b)^2$

$$b>(1-b)^3 \quad \text{i.e. } b>0{\cdot}3177.$$

This problem really depends on *what* you consider to be taking a handful at random. The set of conkers from which the handful is chosen is of unspecified size, but is not infinite. If there are 2N conkers in the bag and it is equally likely that any number 1, 2, 3, . . . , 2N is chosen, the chance of an odd number is clearly $\frac{1}{2}$. If there are 2N+1 conkers, the chance of an odd number is $(2N+1)/(2N+1)$, which is slightly greater than $\frac{1}{2}$. Not knowing whether there are 2N or 2N+1, are we justified in assuming that each is equally likely, considering that the integers start with an odd number, namely unity ? This is, in a sense, a matter of opinion, but basically depends on what we are prepared to assume about the number of conkers which are likely to be selected for a 'bagful'. If odd and even numbers *are* equally likely, then it is true that the chances are slightly in favour of an odd number in the handful.

Arithmetical

Solutions_____________________________________K/S

K1 /S

Now 7, 11, and 13 are all factors of 1,001, or any multiple of this number,

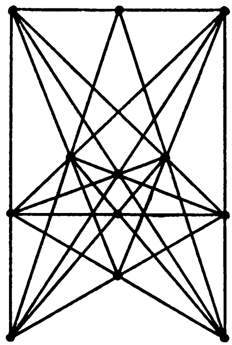
i.e.	2,002
	3,003
	11,011
	111,111
	999,999
	1001,000

Taking the number in question:—

1064893	
1001000	subtract nearest lower multiple of 1001
63893	
63063	subtract nearest lower multiple of 1001
830	

By inspection 830 does not divide by 7, 11, or 13.
Therefore neither does 1064893.

K2/S



The line of four trees has been discounted.

K3/S

```
124)121283l6(97809
    1116
    ----
     968
     868
     ----
     1003
      992
     ----
       1116
       1116
       ----
       ----
       ----
```

K4/S

```
667334)7752341(11·6168830000
       667334
       1079001
        667334
        4116670
        4004004
         1126660
          667334
          4593260
          4004004
           5892560
           5338672
            5538880
            5338672
             2002080
             2002002
                 780000
                 667334
                 112666
```

K5/S

Suppose we want a dozen.

Multiply together all the primes up to 12.

Then add 2, 3, 4,........12 to the result.

This will be the required series

i.e. $2 \times 3 \times 5 \times 7 \times 11 = 2{,}310$

$\therefore$ Series 2,312, 2,313, 2,314,........2322.

Every even number is divisible by 2

$\therefore$ a series of consecutive prime numbers cannot be formulated.

K6/S

The volume of the containers are as the cube of their radii. If d is the diameter of the smallest and the thickness of the wall is $\frac{1}{2}$ unit, the diameters of the containers are d, $d+1$, $d+2$, $d+3$ and

$$d^3+(d+1)^3+(d+2)^3=(d\ 3)^3$$
$$\text{giving } d^3-6d-9=0$$
$$\text{therefore } d=3.$$
$$\text{Since } 3^3+4^3+5^3=6^3.$$

If the walls are all made 1″ thick, then the containers are 6″, 8″, 10″ and 12″.

K7/S

7 digits to choose 4 $=\binom{7}{4}=7.6.5.4=840.$

Omitting digit 6 $=\binom{6}{4}=6.5.4.3=360$

$\therefore$ 840—360 numbers contain 6.

By upturning the 6, 840—360 numbers contain 9.

$\therefore$ total number of cases $=840+480=1320$.

Total combination of 7 digits having 1 in units column is

$$\binom{6}{3}=6.5.4.=120.$$

Of these $5.4.3.=60$ do not contain a 6. Therefore 60 do! Therefore 60 contain 9.

Total No. of 4-figured numbers ending in 1 is 180

„ „ „ „ „ 2 180

etc.

„ „ „ „ „ 6 and 9 120

Total of units column $=180\ (1+2+3+4+5+8)$
$+120\ (6+9)=5{,}940.$

The above also holds for tens, hundreds, etc.

Therefore grand total $=5{,}940+59{,}400+594{,}000+5{,}940{,}000$
$=6{,}599{,}340.$

K8 /S

If two numbers are multiplied together the digital root of the answer is the same as that obtained by multiplying together the digital roots of the two figures concerned. (The digital root is the sum of all the integers in a number).

Now every number has a digital root of from 1 to 9.

In the case of cubes:—

1^3	gives	a digital	root of	1
2^3	,,	,,	,,	8
3^3	,,	,,	,,	9
4^3	,,	,,	,,	1
5^3	,,	,,	,,	8
6^3	,,	,,	,,	9

etc.

K9 /S

$$\begin{array}{lr}
 &)\overline{194481}(441 \\
 & 16 \\
84 & \overline{344} \\
 & 336 \\
881 & \overline{881} \\
 & 881 \\
 & \overline{---}
\end{array}$$

K10 /S

The solution runs as follows:—

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 1, \text{ say, without loss of generality.}$$

Let $A = 3\alpha$, $B = 3\beta$, $C = 3\gamma$.

Then $\dfrac{YC}{\sin \alpha} = \dfrac{AC}{\sin(\alpha + \gamma)}$

$$\mathrm{YC} = \frac{\sin\alpha \sin 3\beta}{\sin(\alpha+\gamma)} = \frac{\sin\alpha\sin(2\pi - 3\alpha - 3\gamma)}{\sin(\alpha+\gamma)}$$

$$= \sin\alpha . \frac{\sin 3(\alpha+\gamma)}{\sin(\alpha+\gamma)}$$

$$= \sin\alpha . \frac{\sin(\alpha+\gamma)\cos 2(\alpha+\gamma) + \cos(\alpha+\gamma)\sin 2(\alpha+\gamma)}{\sin(\alpha+\gamma)}$$

$$= \sin\alpha\,[\cos 2(\alpha+\gamma) + 2\cos^2(\alpha+\gamma)]$$

$$= \sin\alpha\,[1 + \cos 2(\alpha+\gamma)]$$

$$= 2\sin\alpha\,[\tfrac{1}{2} + \cos 2(\alpha+\gamma)]$$

$$= 2\sin\alpha\left[\cos\frac{\pi}{3} + \cos 2(\alpha+\gamma)\right]$$

$$= 4\sin\alpha\left[\cos(\alpha+\gamma+\frac{\pi}{6})\cos(\alpha+\gamma-\frac{\pi}{6})\right]$$

$$= 4\sin\alpha\cos(\beta+\frac{\pi}{6})\sin\beta$$

$$\mathrm{XY}^2 = \mathrm{YC}^2 + \mathrm{XC}^2 - 2\mathrm{YC}.\ \mathrm{XC}\cos\gamma$$

$$\frac{\mathrm{XY}^2}{16} = \sin^2\alpha\sin^2\beta\cos^2(\beta+\frac{\pi}{6}) + \sin^2\alpha\sin^2\beta\cos^2(\alpha+\frac{\pi}{6})$$

$$-2\sin^2\alpha\sin^2\beta\cos\gamma\cos(\beta+\frac{\pi}{6})\cos(\alpha+\frac{\pi}{6})$$

$$= \sin^2\alpha\sin^2\beta\left[\cos^2(\beta+\frac{\pi}{6}) + \cos^2(\alpha+\frac{\pi}{6}) - 2\cos\gamma\right.$$

$$\left.\cos(\beta+\frac{\pi}{6})\cos(\alpha+\frac{\pi}{6})\right]$$

Consider term in square brackets

$$\gamma = \frac{2\pi}{3} - \alpha - \beta$$

$$\cos\gamma = \cos\left[2\gamma\pi - (\alpha+\frac{\pi}{6}) - (\beta+\frac{\pi}{6})\right]$$

$$= \cos\left\{(\alpha+\frac{\pi}{6}) + (\beta+\frac{\pi}{6})\right\}$$

Put $\lambda = \alpha + \frac{\pi}{6}$; $\mu = \beta + \frac{\pi}{6}$

$$
\begin{aligned}
\text{Term} &= \cos^2\lambda + \cos^2\mu - 2\cos(\lambda+\mu)\cos\lambda\cos\mu \\
&= \cos^2\lambda + \cos^2\mu - 2\cos\lambda\cos\mu\,(\cos\lambda\cos\mu - \sin\lambda\sin\mu) \\
&= \cos^2\lambda + \cos^2\mu - 2\cos^2\lambda\cos^2\mu + 2\cos\lambda\sin\lambda\cos\mu\sin\mu \\
&= \cos^2\lambda\,(1-\cos^2\mu) + \cos^2\mu\,(1-\cos^2\lambda) + \tfrac{1}{2}\sin 2\lambda\sin 2\mu \\
&= \cos^2\lambda\sin^2\mu + \cos^2\mu\sin^2\lambda + \tfrac{1}{2}\sin 2\lambda\sin 2\mu \\
&= \frac{1+\cos 2\lambda}{2}\cdot\frac{1-\cos 2\mu}{2} + \frac{1+\cos 2\mu}{2}\cdot\frac{1-\cos 2\lambda}{2} \\
&\qquad\qquad + \tfrac{1}{2}\sin 2\lambda\sin 2\mu \\
&= \tfrac{1}{2}\,[1 + \sin 2\lambda\sin 2\mu - \cos 2\mu\cos 2\lambda] \\
&= \tfrac{1}{2}\,[1 - \cos(\lambda+\mu)] = \sin^2(\lambda+\mu) \\
&= \sin^2\left(\alpha+\beta+\frac{\pi}{3}\right) = \sin^2(\pi-\gamma) = \sin^2\gamma
\end{aligned}
$$

Therefore $XY^2 = 16\sin^2\alpha\sin^2\beta\sin^2\gamma$
and hence by symmetry all sides are equal.