

12. RATIO AND PROPORTION

IMPORTANT FACTS AND FORMULAE

I. RATIO: The ratio of two quantities a and b in the same units, is the fraction a/b and we write it as $a:b$. In the ratio $a:b$, we call a as the **first term or antecedent** and b , the **second term or consequent**.

Ex. The ratio $5:9$ represents $5/9$ with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Ex. $4:5 = 8:10 = 12:15$ etc. Also, $4:6 = 2:3$.

2. PROPORTION: *The equality of two ratios is called proportion.*

If $a:b = c:d$, we write, $a:b::c:d$ and we say that a, b, c, d are in proportion. Here a and d are called extremes, while b and c are called mean terms.

Product of means = Product of extremes.

Thus, $a:b::c:d \Leftrightarrow (b \times c) = (a \times d)$.

3. (i) Fourth Proportional: If $a:b = c:d$, then d is called the fourth proportional to a, b, c .

(ii) Third Proportional: If $a:b = b:c$, then c is called the third proportional to a and b .

(iii) Mean Proportional: Mean proportional between a and b is *square root of ab*

4. (i) COMPARISON OF RATIOS:

We say that $(a:b) > (c:d) \Leftrightarrow (a/b) > (c/d)$.

(ii) COMPOUNDED RATIO:

The compounded ratio of the ratios $(a:b), (c:d), (e:f)$ is $(ace:bdf)$

5. (i) Duplicate ratio of $(a:b)$ is $(a^2:b^2)$.

(ii) Sub-duplicate ratio of $(a:b)$ is $(\sqrt{a}:\sqrt{b})$.

(iii) Triplicate ratio of $(a:b)$ is $(a^3:b^3)$.

(iv) Sub-triplicate ratio of $(a:b)$ is $(a^{1/3}:b^{1/3})$.

(v) If $(a/b) = (c/d)$, then $((a+b)/(a-b)) = ((c+d)/(c-d))$ **(Componendo and dividendo)**

6. VARIATION:

(i) We say that x is directly proportional to y , if $x = ky$ for some constant k and we write, $x \propto y$.

(ii) We say that x is inversely proportional to y , if $xy = k$ for some constant k and we write, $x \propto (1/y)$

SOLVED PROBLEMS

Ex. 1. If $a : b = 5 : 9$ and $b : c = 4 : 7$, find $a : b : c$.

Sol. $a:b=5:9$ and $b:c=4:7 \Rightarrow (4 \times 9/4) : (7 \times 9/4) = 9:63/4$
 $a:b:c = 5:9:63/4 = 20:36:63$.

Ex 2. Find:

- (i) *the fourth proportional to 4, 9, 12;*
- (ii) *the third proportional to 16 and 36;*
- iii) *the mean proportional between 0.08 and 0.18.*

Sol.

i) Let the fourth proportional to 4, 9, 12 be x.

Then, $4 : 9 :: 12 : x \Leftrightarrow 4 \times x = 9 \times 12 \Leftrightarrow X = (9 \times 12)/4 = 27$;
Fourth proportional to 4, 9, 12 is 27.

(ii) Let the third proportional to 16 and 36 be x.

Then, $16 : 36 :: 36 : x \Leftrightarrow 16 \times x = 36 \times 36 \Leftrightarrow x = (36 \times 36)/16 = 81$
Third proportional to 16 and 36 is 81.

(iii) Mean proportional between 0.08 and 0.18

$$\sqrt{0.08 \times 0.18} = \sqrt{8/100 \times 18/100} = \sqrt{144/(100 \times 100)} = 12/100 = 0.12$$

Ex. 3. If $x : y = 3 : 4$, find $(4x + 5y) : (5x - 2y)$.

Sol. $X/Y=3/4 \Leftrightarrow (4x+5y)/(5x+2y) = (4(x/y)+5)/(5(x/y)-2) = (4(3/4)+5)/(5(3/4)-2)$
 $= (3+5)/(7/4) = 32/7$

Ex. 4. Divide Rs. 672 in the ratio 5 : 3.

Sol. Sum of ratio terms = $(5 + 3) = 8$.

First part = Rs. $(672 \times (5/8))$ = Rs. 420; Second part = Rs. $(672 \times (3/8))$ = Rs. 252.

Ex 5. Divide Rs. 1162 among A, B, C in the ratio 35 : 28 : 20.

Sol. Sum of ratio terms = $(35 + 28 + 20) = 83$.

A's share = Rs. $(1162 \times (35/83)) =$ Rs. 490; B's share = Rs. $(1162 \times (28/83)) =$ Rs. 392;

C's share = Rs. $(1162 \times (20/83)) =$ Rs. 280.

Ex 6. A bag contains 50 p, 25 P and 10 p coins in the ratio 5: 9: 4, amounting to Rs. 206. Find the number of coins of each type.

Sol. Let the number of 50 p, 25 P and 10 p coins be $5x$, $9x$ and $4x$ respectively.

$$(5x/2) + (9x/4) + (4x/10) = 206 \Leftrightarrow 50x + 45x + 8x = 4120 \Leftrightarrow 103x = 4120 \Leftrightarrow x = 40.$$

Number of 50 p coins = $(5 \times 40) = 200$; Number of 25 p coins = $(9 \times 40) = 360$;

Number of 10 p coins = $(4 \times 40) = 160$.

Ex 7. A mixture contains alcohol and water in the ratio 4 : 3. If 5 litres of water is added to the mixture, the ratio becomes 4: 5. Find the quantity of alcohol in the given mixture

Sol. Let the quantity of alcohol and water be $4x$ litres and $3x$ litres respectively

$$4x/(3x+5) = 4/5 \Leftrightarrow 20x = 4(3x+5) \Leftrightarrow 8x = 20 \Leftrightarrow x = 2.5$$

Quantity of alcohol = (4×2.5) litres = 10 litres.