

## 15. TIME AND WORK

### IMPORTANT FACTS AND FORMULAE

1. If A can do a piece of work in  $n$  days, then A's 1 day's work =  $(1/n)$ .
2. If A's 1 day's work =  $(1/n)$ , then A can finish the work in  $n$  days.
3. A is thrice as good a workman as B, then:  
Ratio of work done by A and B = 3 : 1.  
Ratio of times taken by A and B to finish a work = 1 : 3.

### SOLVED EXAMPLES

**Ex. 1. Worker A takes 8 hours to do a job. Worker B takes 10 hours to do the same Job. How long should it take both A and B, working together but independently, to do the same job? (IGNOU, 2003)**

**Sol.** A's 1 hour's work =  $1/8$

B's 1 hour's work =  $1/10$

(A + B)'s 1 hour's work =  $(1/8) + (1/10) = 9/40$

Both A and B will finish the work in  $40/9$  days.

**Ex. 2. A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work? (Bank P.O. 2003)**

**Sol.** (A + B)'s 1 day's work =  $(1/4)$ . A's 1 day's work =  $(1/12)$ .

B's 1 day's work =  $((1/4) - (1/12)) = (1/6)$

Hence, B alone can complete the work in 6 days.

**Ex. 3. A can do a piece of work in 7 days of 9 hours each and B can do it in 6 days of 7 hours each. How long will they take to do it, working together 8 hours a day?**

**Sol.** A can complete the work in  $(7 \times 9) = 63$  hours.

B can complete the work in  $(6 \times 7) = 42$  hours.

A's 1 hour's work  $= (1/63)$  and B's 1 hour's work  $= (1/42)$

$(A + B)$ 's 1 hour's work  $= (1/63) + (1/42) = (5/126)$

Both will finish the work in  $(126/5)$  hrs.

Number of days. of  $(42/5)$  hrs each  $= (126 \times 5) / (5 \times 42) = 3$  days

**Ex. 4. A and B can do a piece of work in 18 days; Band C can do it in 24 days A and C can do it in 36 days. In how many days will A, Band C finish it together and separately?**

**Sol.**  $(A + B)$ 's 1 day's work  $= (1/18)$   $(B + C)$ 's 1 day's work  $= (1/24)$

and  $(A + C)$ 's 1 day's work  $= (1/36)$

Adding, we get:  $2(A + B + C)$ 's 1 day's work  $= (1/18 + 1/24 + 1/36)$

$$= 9/72 = 1/8$$

$(A + B + C)$ 's 1 day's work  $= 1/16$

Thus, A, Band C together can finish the work in 16 days.

Now, A's 1 day's work  $= [(A + B + C)$ 's 1 day's work] -  $[(B + C)$ 's 1 day work:

$$= (1/16 - 1/24) = 1/48$$

A alone can finish the work in 48 days.

Similarly, B's 1 day's work  $= (1/16 - 1/36) = 5/144$

B alone can finish the work in  $144/5 = 28 \frac{4}{5}$  days

And C's 1 day work  $= (1/16 - 1/18) = 1/144$

Hence C alone can finish the work in 144 days.

**Ex. 6. A is twice as good a workman as B and together they finish a piece in 18 days. In how many days will A alone finish the work?**

**Sol.**  $(A$ 's 1 day's work): $(B$ 's 1 days work)  $= 2 : 1$ .

$(A + B)$ 's 1 day's work  $= 1/18$

Divide  $\frac{1}{18}$  in the ratio 2 : 1.

$$\therefore \text{A's 1 day's work} = \left(\frac{1}{18} \times \frac{2}{3}\right) = \frac{1}{27}$$

Hence, A alone can finish the work in 27 days.

**Ex. 6. A can do a certain job in 12 days. B is 60% more efficient than A. How many days does B alone take to do the same job?**

**Sol.** Ratio of times taken by A and B = 160 : 100 = 8 : 5.

Suppose B alone takes x days to do the job.

$$\text{Then, } 8 : 5 :: 12 : x = 8x = 5 \times 12 = x = 7 \frac{1}{2} \text{ days.}$$

**Ex. 7. A can do a piece of work in 80 days. He works at it for 10 days B alone finishes the remaining work in 42 days. In how much time will A and B working together, finish the work?**

$$\text{Sol. Work done by A in 10 days} = \left(\frac{1}{80} \times 10\right) = \frac{1}{8}$$

$$\text{Remaining work} = \left(1 - \frac{1}{8}\right) = \frac{7}{8}$$

Now,  $\frac{7}{8}$  work is done by B in 42 days.

Whole work will be done by B in  $(42 \times \frac{8}{7}) = 48$  days.  
A's 1 day's work =  $\frac{1}{80}$  and B's 1 day's work =  $\frac{1}{48}$

$$(A+B)\text{'s 1 day's work} = \left(\frac{1}{80} + \frac{1}{48}\right) = \frac{8}{240} = \frac{1}{30}$$

Hence, both will finish the work in 30 days.

**Ex. 8. A and B undertake to do a piece of work for Rs. 600. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days. Find the share of each.**

$$\text{Sol :C's 1 day's work} = \frac{1}{3} - \left(\frac{1}{6} + \frac{1}{8}\right) = \frac{1}{24}$$

$$A : B : C = \text{Ratio of their 1 day's work} = \frac{1}{6} : \frac{1}{8} : \frac{1}{24} = 4 : 3 : 1.$$

$$\text{A's share} = \text{Rs. } (600 \times \frac{4}{8}) = \text{Rs. } 300, \text{ B's share} = \text{Rs. } (600 \times \frac{3}{8}) = \text{Rs. } 225.$$

$$\text{C's share} = \text{Rs. } [600 - (300 + 225)] = \text{Rs. } 75.$$

**Ex. 9. A and B working separately can do a piece of work in 9 and 12 days respectively, If they work for a day alternately, A beginning, in how many days, the work will be completed?**

$$(A + B)\text{'s 2 days' work} = \left(\frac{1}{9} + \frac{1}{12}\right) = \frac{7}{36}$$

$$\text{Work done in 5 pairs of days} = \left(5 \times \frac{7}{36}\right) = \frac{35}{36}$$

$$\text{Remaining work} = \left(1 - \frac{35}{36}\right) = \frac{1}{36}$$

On 11th day, it is A's turn.  $\frac{1}{9}$  work is done by him in 1 day.

$\frac{1}{36}$  work is done by him in  $(9 \times \frac{1}{36}) = \frac{1}{4}$  day

Total time taken =  $(10 + \frac{1}{4})$  days =  $10 \frac{1}{4}$  days.

**Ex 10 .45 men can complete a work in 16 days. Six days after they started working, 30 more men joined them. How many days will they now take to complete the remaining work?**

$(45 \times 16)$  men can complete the work in 1 day.

1 man's 1 day's work =  $\frac{1}{720}$

45 men's 6 days' work =  $(\frac{1}{16} \times 6) = \frac{3}{8}$

Remaining work =  $(1 - \frac{3}{8}) = \frac{5}{8}$

75 men's 1 day's work =  $\frac{75}{720} = \frac{5}{48}$

Now,  $\frac{5}{48}$  work is done by them in 1 day.

$\frac{5}{8}$  work is done by them in  $(\frac{48}{5} \times \frac{5}{8}) = 6$  days.

Ex:11. 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days. In how many days can 2 men and 1 boy do the work?

Soln: Let 1 man's 1 day's work = x and 1 boy's 1 day's work = y.

Then,  $2x + 3y = \frac{1}{10}$  and  $3x + 2y = \frac{1}{8}$

Solving, we get:  $x = \frac{7}{200}$  and  $y = \frac{1}{100}$

$(2 \text{ men} + 1 \text{ boy})$ 's 1 day's work =  $(2 \times \frac{7}{200} + 1 \times \frac{1}{100}) = \frac{16}{200} = \frac{2}{25}$

So, 2 men and 1 boy together can finish the work in  $\frac{25}{2} = 12 \frac{1}{2}$  days