23. LOGARITHMS

IMPORTANT FACTS AND FORMULAE

I. Logarithm: If a is a positive real number, other than 1 and a^m = X, then we write: **m = $\boldsymbol{log_a}$ x** and we say that the value of log x to the base *a* is m.

Example:

(i) $10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$

(ii) $2^{-3} = 1/8 \Rightarrow \log_2 1/8 = -3$

(iii) $3^4 = 81 \Rightarrow \log_3 81=4$

(iiii) $(.1)^2 = .01 \Rightarrow \log_{(.1)} .01 = 2.$

II. Properties of Logarithms:

1. $\log_a(xy) = \log_a x + \log_a y$

2. $\log_a (x/y) = \log_a x - \log_a y$

3. $\log_x x=1$

4. $\log_a 1 = 0$

5. $\log_a(x^p)=p(\log_a x)$ 1

6. $\log_a x = 1/\log_x a$

7. $\log_a x = \log_b x/\log_b a=\log\_x/\log a.$

Remember: When base is not mentioned, it is taken as 10.

II. Common Logarithms:

Logarithms to the base 10 are known as common logarithms.

III. The logarithm of a number contains two parts, namely *characteristic* and *mantissa.*

Characteristic: The integral part of the logarithm of a number is called its ***characteristic.***

Case I: When the number is greater than 1.

In this case, the characteristic is one less than the number of digits in the left of the decimal point in the given number.

Case II: When the number is less than 1.

In this case, the characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and it is negative.

Instead of - 1, - 2, etc. we write, $\bar{1}$ (one bar), $\bar{2}$ (two bar), etc.

Example:

Number	*Characteristic*	*Number*	*Characteristic*
348.25	2	0.6173	$\bar{1}$
46.583	1	0.03125	$\bar{2}$
9.2193	0	0.00125	$\bar{3}$

Mantissa: The decimal part of the logarithm of a number is known is its ***mantissa.*** For mantissa, we look through log table.

SOLVED EXAMPLES

1.Evaluate:

(1)$\mathbf{log}_3$ 27

(2)$\mathbf{log}_7$ (1/343)

(3)$\mathbf{log}_{100(0.01)}$

SOLUTION:

(1) let $\log_3 27 = 3^3$ or n=3.

ie, $\log_3 27 = 3$.

(2) Let $\log_7 (1\backslash 343) = n$.

Then ,$7^n = 1/343$

$= 1/7^3$

n = -3.

ie,

$\log_7(1\backslash 343) = -3$.

(3) let $\log_{100}(0.01) = n$.

Then,. (100) = 0.01 = 1 /100=100 $^{-1}$ 0r n=-1

EX.2. evaluate

(i) $\log_7 1=0$ (ii)$\log_{34} 34$ (iii)$36^{\log}{}_6\ 4$

solution:

i) we know that $\log_a 1=0$,so $\log_7 1=0$.

ii) we know that $\log_a a=1$,so $\log_{34} 34=0$.

iii) We know that $a^{\log}{}_6\ x = x$.

. now $36^{\log}{}_6\ 4 = (6^2)^{\log}{}_6\ {}^4 = 6^{\log}{}_6{}^{(16)=16.}$

Ex.3.if $\log_{\sqrt{8}}$ x=3 (1/3), find the value of x.

$\log_{\sqrt{8}}$ x=10/3 ,x=$(\sqrt{8})^{10/3}=(2^{3/2})^{10/3}=2^{(3/2*10/3)}=2^5$=32.

Ex.4:Evaluate: (i) $\log_5 3*\log_{27} 25$ (ii) log 27 –$\log_{27}$ 9

(i)$\log_5{}^3 * \log_{27} 25$=(log 3/log 5)*(log 25/log 27)
=(log 3/log 5)*(log 5^2*log3^3)
=(log 3/log 5)*(2log5/3log3)
=2/3

(ii)Let $\log_9 27$=n
Then,
$9^n = 27 \Leftrightarrow 3^{2n} = 3^3 \Leftrightarrow 2n=3 \Leftrightarrow n=3/2$
Again, let $\log_{27} 9$=m
Then,
$27^m = 9 \Leftrightarrow 3^{3m} = 3^2 \Leftrightarrow 3m=2 \Leftrightarrow m=2/3$

- ⇨ $\log_9 27 - \log_{27} 9$=(n-m)=(3/2-2/3)=5/6

Ex 5. Simplify :(log 75/16-2 log 5/9+log 32/243)
Sol: log 75/16-2 log 5/9+log 32/243
= log 75/16-log$(5/9)^2$+log32/243
= log 75/16-log25/81+log 32/243
= log(75/16*32/243*81/25)=log 2

Ex. 6.Find the value of x which satisfies the relation
Log_{10} 3+$\log_{10}$ (4x+1)=$\log_{10}$ (x+1)+1
Sol: $\log_{10}$ 3+$\log_{10}$ (4x+1)=$\log_{10}$ (x+1)+1
Log_{10} 3+$\log_{10}$ (4x+1)=$\log_{10}$ (x+1)+$\log_{10}$ (x+1)+$\log_{10}$ 10
Log_{10} (3(4x+1))=$\log_{10}$ (10(x+1))
=3(4x+1)=10(x+1)=12x+3
=10x+10
=2x=7=x=7/2

Ex. 7.Simplify:[1/$\log_{xy(}$xyz)+1/$\log_{yz}$(xyz)+1/$\log_{zx}$(xyz)]
Given expression: $\log_{xyz}$ xy+ $\log_{xyz}$ yz+ $\log_{xyz}$ zx
=$\log_{xyz}$ (xy*yz*zx)=$\log_{xyz}$ $(xyz)^2$
2$\log_{xyz}$(xyz)=2*1=2

Ex.8.If $\log_{10}$ 2=0.30103,find the value of $\log_{10}$ 50.
Soln. $\log_{10}$ 50=$\log_{10}$ (100/2)=$\log_{10}$ 100-$\log_{10}$ 2=2-0.30103=1.69897.

Ex 9.If log 2=0.3010 and log 3=0.4771,find the values of:

i) log 25 ii)log 4.5

Soln.

i) log 25=log(100/4)=log 100-log 4=2-2log 2=(2-2*.3010)=1.398.

ii) log 4.5=log(9/2)=log 9-log 2=2log 3-log 2
=(2*0.4771-.3010)=.6532

Ex.10. If log 2=.30103,find the number of digits in 2^{56}.

Soln. log 2^{56} =56log2=(56*0.30103)=16.85768.

Its characteristics is 16.

Hence,the number of digits in 2^{56} is 17