24 .AREA

FUNDEMENTAL CONCEPTS

I.RESULTS ON TRIANGLES:
1.Sum of the angles of a triangle is 180 degrees.

2.Sum of any two sides of a triangle is greater than the third side.

3.Pythagoras theorem:
In a right angle triangle,
(Hypotenuse)^2 = (base)^2 + (Height)^2

4.The line joining the midpoint of a side of a triangle to the opposite vertex is called the

MEDIAN
5.The point where the three medians of a triangle meet is called **CENTROID**.
Centroid divides each of the medians in the ratio 2:1.

6.In an isosceles triangle, the altitude from the vertex bi-sects the base

7.The median of a triangle divides it into two triangles of the same area.

8.Area of a triangle formed by joining the midpoints of the sides of a given triangle is one-fourth of the area of the given triangle.

II.RESULTS ON QUADRILATERALS:

1. The diagonals of a parallelogram bisects each other .
2. Each diagonal of a parallelogram divides it into two triangles of the same area
3. The diagonals of a rectangle are equal and bisect each other.
4. The diagonals of a square are equal and bisect each other at right angles.
5. The diagonals of a rhombus are unequal and bisect each other at right angles.
6. A parallelogram and a rectangle on the same base and between the same parallels are equal in area.
7. Of all the parallelograms of a given sides , the parallelogram which is a rectangle has the greatest area.

IMPORTANT FORMULAE

I.1.Area of a rectangle=(length*breadth)
Therefore length = (area/breadth) and breadth=(area/length)
2.Perimeter of a rectangle = 2*(length+breadth)
II.Area of a square = (side)^2 =1/2(diagonal)^2
III Area of four walls of a room = 2*(length + breadth)*(height)

IV 1.Area of the triangle=1/2(base*height)

2. Area of a triangle = (s*(s-a)(s-b)(s-c))^(1/2), where a,b,c are the sides of a triangle and s= ½(a+b+c)

3.Area of the equilateral triangle =((3^1/2)/4)*(side)^2

4.Radius of incircle of an equilateral triangle of side a=a/2(3^1/2)

5.Radius of circumcircle of an equilateral triangle of side a=a/(3^1/2)

6.Radius of incircle of a triangle of area del and semiperimeter S=del/S

V.1.Area of the parellogram =(base *height)

2.Area of the rhombus=1/2(product of the diagonals)

3.Area of the trapezium=1/2(size of parallel sides)*distance between them

VI 1.Area of a circle =pi*r^2,where r is the radius

2. Circumference of a circle = 2∏R.

3. Length of an arc = 2∏Rθ/(360) where θ is the central angle

4. Area of a sector = (1/2) (arc x R) = pi*R^2*θ/360.

VII. 1. Area of a semi-circle = (pi)*R^2.

2. Circumference of a semi-circle = (pi)*R.

SOLVED EXAMPLES

Ex.1. One side of a rectangular field is 15 m and one of its diagonals is 17 m. Find the area of the field.

Sol. Other side = $((17)^2 - (15)^{2)(1/2}) = (289- 225)^{(1/2)} = (64)^{(1/2)} = 8$ m.
Area = (15 x 8) m^2 = 120 m^2.

Ex. 2. A lawn is in the form of a rectangle having its sides in the ratio 2: 3. The area of the lawn is (1/6) hectares. Find the length and breadth of the lawn.

Sol. Let length = 2x metres and breadth = 3x metre.

Now, area = (1/6)x 1000 m^2 = 5000/3m^2

So, 2x * *3x* = 5000/3 <=> x^2 = 2500/9 <=> *x* = 50/3
therefore Length = 2x = (100/3) m = 33(1/3) m and Breadth = *3x* = 3(50/3) m = 50m.

Ex. 3. Find the cost of carpeting a room 13 m long and 9 m broad with a carpet 75 cm wide at the rate of Rs. 12.40 per square metre.
Sol. Area of the carpet = Area of the room = (13 * 9) m^2 = 117 m^2.
Length of the carpet = (area/width) = 117 *(4/3) m = 156 m.
Therefore Cost of carpeting = Rs. (156 * 12.40) = Rs. 1934.40.

Ex. 4. If the diagonal of a rectangle is 17 cm long and its perimeter is 46 cm, find the area of the rectangle. .

Sol. Let length = x and breadth = *y*. Then,
$2(x + y) = 46$ or $x + y = 23$ and $x^2 + y^2 = (17)^2 = 289$.
Now, $(x + y)^2 = (23)^2 \Leftrightarrow (x^2 + y^2) + 2xy = 529 \Leftrightarrow 289 + 2xy = 529 \Leftrightarrow xy=120$
Area = $xy = 120\ cm^2$.

Ex. 5. The length of a rectangle is twice its breadth. If its length is decreased by 5 cm and breadth is increased by 5 cm, the area of the rectangle is increased by 75 sq. cm. Find the length of the rectangle.

Sol. Let breadth = x. Then, length = *2x*. Then,
$(2x - 5)(x + 5) - 2x * x = 75 \Leftrightarrow 5x - 25 = 75 \Leftrightarrow x = 20$.
:. Length of the rectangle = 20 cm.

Ex. 6. In measuring the sides of a rectangle, one side is taken 5% in excess, and the other 4% in deficit. Find the error percent in the area calculated from these measurements. (M.B.A. 2003)
Sol. Let x and *y* be the sides of the rectangle. Then, Correct area = xy.
Calculated area = (105/100)*x * (96/100)*y = (504/500)(xy)
Error In measurement = (504/500)xy- *xy* = (4/500)xy
Error % = [(4/500)*xy* *(1/xy) *100] % = (4/5) % = 0.8%.

Ex. 7. A rectangular grassy plot 110 m. by 65 m has a gravel path 2.5 m wide all round it on the inside. Find the cost of gravelling the path at 80 paise per sq. metre.

Sol. Area of the plot = $(110 \times 65)\ m^2 = 7150\ m^2$
Area of the plot excluding the path = $[(110 - 5) * (65 - 5)]\ m^2 = 6300\ m^2$.
Area of the path = $(7150 - 6300)\ m^2 = 850\ m^2$.

Cost of gravelling the path = Rs.850 * (80/100)= Rs. 680

Ex. 8. The perimeters of two squares are 40 cm and 32 cm. Find the perimeter of a third square whose area is equal to the difference of the areas of the two squares. (S.S.C. 2003)
Sol. Side of first square = (40/4) = 10 cm;
Side of second square = (32/4)cm = 8 cm.
Area of third square = $[(10)^2 - (8)^2]\ cm^2 = (100 - 64)\ cm^2 = 36\ cm^2$.
Side of third square = $(36)^{(1/2)}$ cm = 6 cm.
Required perimeter = (6 x 4) cm = 24 cm.

Ex. 9. A room 5m 55cm long and 3m 74 cm broad is to be paved with square tiles. Find the

least number of square tiles required to cover the floor.

Sol. Area of the room = (544 x 374) cm^2.

Size of largest square tile = H.C.F. of 544 cm and 374 cm = 34 cm.

Area of 1 tile = (34 x 34) cm^2.

Number of tiles required =(544*374)/(34*34)=176

Ex. 10. Find the area of a square, one of whose diagonals is 3.8 m long.

Sol. Area of the square = (1/2)* (diagonal) ^{2} = [(1/2)*3.8*3.8]m^2 = 7.22 m^2.

Ex. 11. The diagonals of two squares are in the ratio of 2 : 5. Find the ratio of their areas. (Section Officers', 2003)

Sol. Let the diagonals of the squares be 2x and 5x respectively.

Ratio of their areas = (1/2)*$(2x)^2$:(1/2)*$(5x)^2$ = $4x^2$: $25x^2$ = 4 : 25.

Ex.12. If each side of a square is increased by 25%, find the percentage change in its area.

Sol. Let each side of the square be *a*. Then, area = a^2.

New side =(125a/100) =(5a/4). New area = $(5a/4)^2$ =($25a^2$)/16.

Increase in area = ((25 a^2)/16)-a^2 =($9a^2$)/16.

Increase% = [(($9a^2$)/16)*(1/a^2)*100] % = 56.25%.

Ex. 13. If the length of a certain rectangle is decreased by 4 cm and the width is increased by 3 cm, a square with the same area as the original rectangle would result. Find the perimeter of the original rectangle.

Sol. Let x and y be the length and breadth of the rectangle respectively.

Then, x - 4 = y + 3 or x - y = 7 ----(i)

Area of the rectangle =xy; Area of the square = (x - 4) (*y* + 3)

(x - 4) (y + 3) =xy <=> 3x - 4*y* = 12 ----(ii)

Solving *(i)* and *(ii)*, we get x = 16 and *y* = 9.

Perimeter of the rectangle = 2 (x + *y)* = [2 (16 + 9)] cm = 50 cm.

Ex. 14. A room is half as long again as it is broad. The cost of carpeting the at Rs. 5 per sq. m is Rs. 270 and the cost of papering the four walls at Rs. 10 per m^2 is Rs. 1720. If a door and 2 windows occupy 8 sq. m, find the dimensions of the room.

Sol. Let breadth = x metres, length = <u>*3x*</u> metres, height = H metres.

Area of the floor=(Total cost of carpeting)/(Rate/m^2)=(270/5)m^2=54m^2.
x* (3x/2) = 54 <=> x^2 = (54*2/3) = 36 <=> x = 6.
So, breadth = 6 m and length =(3/2)*6 = 9 m.
Now, papered area = (1720/10)m^2 = 172 m^2.
Area of 1 door and 2 windows = 8 m^2.
Total area of 4 walls = (172 + 8) m^2 = 180 m^2
2*(9+ 6)* H = 180 <=> H = 180/30 = 6 m.

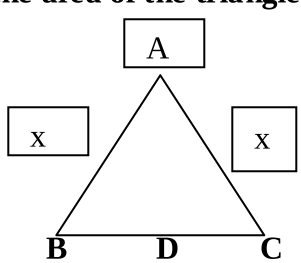
Ex. 15. Find the area of a triangle whose sides measure 13 cm, 14 cm and 15 cm.

Sol. Let a = 13, b = 14 and c = 15. Then, S = (1/2)$(a + b + c)$ = 21.
(s- a) = 8, (s - b) = 7 and (s - c) = 6.
Area = $(s(s-a)(s-b)(s-c))^{(1/2)}$ = $(21*8*7*6)^{(1/2)}$ = 84 cm^2.

Ex. 16. Find the area of a right-angled triangle whose base is 12 cm and hypotenuse is 13cm.
Sol. Height of the triangle = $[(13)^2 - (12)^2]^{(1/2)}$ cm = $(25)^{(1/2)}$ cm = 5 cm.
Its area = (1/2)* Base * Height = ((1/2)*12 * 5) cm^2 = 30 cm^2.

Ex. 17. The base of a triangular field is three times its altitude. If the cost of cultivating the field at Rs. 24.68 per hectare be Rs. 333.18, find its base and height.
Sol. Area of the field = Total cost/rate = (333.18/25.6)hectares = 13.5 hectares
⇔ (13.5 x 10000) m^2 = 135000 m^2.
Let altitude = x metres and base = 3x metres.
Then, (1/2)* $3x * x$ = 135000 <=>x^2= 90000 <=>x = 300.
Base = 900 m and Altitude = 300 m.

Ex. 18. The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. Find the area of the triangle.



Sol. Let ABC be the isosceles triangle and AD be the altitude.
Let AB = AC = x. Then, BC = (32 - $2x$).

Since, in an isosceles triangle, the altitude bisects the base,
so BD = DC = (16 - *x)*.
In *triangle* ADC, AC^2= AD + DC^2=>x^2=(8^2)+$(16\text{-}x)^2$
=>*32x* = 320 =>x= 10.
BC = (32- *2x)* = (32 - 20) cm = 12 cm.
Hence, required area = ((1/2)x*BC * AD) = ((1/2)*12 *10)cm^2 = 60 cm^2.

Ex. 19. Find the length of the altitude of an equilateral triangle of side 3√3 cm.

Sol. Area of the triangle = (√3/4) x *(3 √3)2* = *27 √3.* Let the height be *h*.

Then, (1/2) x 3√3 x *h* = **(**27√3/4) X(2/√3) = 4.5 cm.
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Ex. 20. In two triangles, the ratio of the areas is 4 : 3 and the ratio of their heights is 3 : 4. Find the ratio of their bases.

Sol. Let the bases of the two triangles be x and y and their heights be *3h* and *4h* respectively. Then,
((1/2) X x X 3h)/(1/2) X y X 4h) =4/3 ⇔ x/y =(4/3 X 4/3)=16/9
Required ratio = 16 : 9.

Ex.21. The base of a parallelogram is twice its height. If the area of the parallelogram is 72 sq. cm, find its height.

Sol. Let the height of the parallelogram be x. cm. Then, base = (2x*)* cm.

2x X x =72 ⇔ 2x^2 = 72 ⇔ X ^2=36 ⇔ x=6
Hence, height of the parallelogram = 6 cm.

Ex. 22. Find the area of a rhombus one side of which measures 20 cm and 01 diagonal 24 cm.
Sol. Let other diagonal = 2x cm.
Since diagonals of a rhombus bisect each other at right angles, we have:

(20)2 = (12)2 + (x)2 _ *x* = √(20)2 – (12)2= √256= 16 cm. _I
So, other diagonal = 32 cm.

Area of rhombus = (1/2) x (Product of diagonals) = *((1/2)x* 24 x 32) cm^2 = 384 cm^2

Ex. 23. The difference between two parallel sides of a trapezium is 4 cm. perpendicular distance between them is 19 cm. If the area of the trapezium is 475 find the lengths of the parallel sides. (R.R.B. 2002)
Sol. Let the two parallel sides of the trapezium be *a* em and b em.
Then, *a* - *b* = 4

And, (1/2) x *(a* + *b)* x 19 = 475 ⇔ *(a* + *b)* =((475 x 2)/19) ⇔ *a* + *b* = 50

Solving (i) and *(*ii*)*, we get: $a = 27$, $b = 23$.
So, the two parallel sides are 27 cm and 23 cm.

Ex. 24. Find the length of a rope by which a cow must be tethered in order tbat it may be able to graze an area of 9856 sq. metres. (M.A.T. 2003)

Sol. Clearly, the cow will graze a circular field of area 9856 sq. metres and radius equal to the length of the rope.
Let the length of the rope be R metres.

Then, ∏(R)^2 = (9856 X (7/22)) = 3136⇔ R = 56.

Length of the rope = 56 m.

Ex. 25. The area of a circular field is 13.86 hectares. Find the cost of fencing it at the rate of Rs. 4.40 per metre.

Sol. Area = (13.86 x 10000) m^2= 138600 m^2.

∏(R^2= 138600 ⇔(R)2 = (138600 x (7/22)) ⇔ R = 210 m.

Circumference = 2∏R = (2 x (22/7) x 210) m = 1320 m.
Cost of fencing = Rs. (1320 x 4.40) = Rs. 5808.

Ex. 26. The diameter of the driving wheel of a bus is 140 em. How many revolution, per minute must the wheel make in order to keep a speed of 66 kmph ?

Sol. Distance to be covered in 1 min. = (66 X_1000)/(60) m = 1100 m.

Circumference of the wheel = (2 x (22/7) x 0.70) m = 4.4 m.

Number of revolutions per min. =(1100/4.4) = 250.

Ex, 27. A wheel makes 1000 revolutions in covering a distance of 88 km. Find the radius of the wheel.

Sol. Distance covered in one revolution =((88 X 1000)/1000)= 88m.

2∏R = 88 ⇔ 2 x (22/7) x R = 88 ⇔ R = 88 x (7/44) = 14 m.

Ex, 28. The inner circumference of a circular race track, 14 m wide, is 440 m. Find radius of the outer circle.

Sol . Let inner radius be *r* metres. Then, 2∏r = 440 ⇔ r = (440 x (7/44))= 70 m.

Radius of outer circle = (70 + 14) m = 84 m.

Ex, 29. Two concentric circles form a ring. The inner and outer circumferences of ring are (352/7) m and (518/7) m respectively. Find the width of the ring.

Sol.. Let the inner and outer radii be r and R metres.

Then $2\prod r$ = (352/7) ⇔ r =((352/7) X (7/22) X (1/2))=8m.

2∏R=(528/7) ⇔ R=((528/7) X (7/22) X (1/2))= 12m.

, ', Width of the ring = (R - *r)* = (12 - 8) m = 4 m.

Ex, 30. A sector of 120', cut out from a circle, has an area of (66/7) sq. cm. Find the radius of the circle.

Sol. Let the radius of the circle be *r* cm. Then,

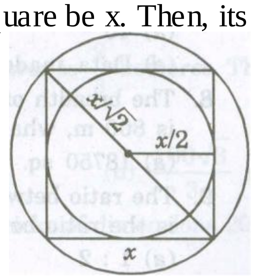
(∏(r)2 θ) /360=(66/7) ⇔ (22/7) X (r) 2 X(120/360)= (66/7)

⇔ (r)2=((66/7) X (7/22) X 3) ⇔ r=3.

Hence, radius = 3 cm.

Ex, 31. Find the ratio of the areas of the incircle and circumcircle of a square.

Sol. Let the side of the square be x. Then, its diagonal = √2 *x*.



Radius of incircle = (x/2)

Radius of circum circle= (√2x/2) =(x/√2)

Required ratio = ((∏ (r)2)/4 : (∏(r)2) /2) = (1/4) : 1/2) = 1 : 2.

Ex. 32. If the radius of a circle is decreased by 50%, find the percentage decrease in its area.

Sol. Let original radius = R. New radius =(50/100) R = (R/2)

Original area=∏(R)2= and new area= ∏ ((R/2))2= (∏ (R)2)/4

Decrease in area =((3∏ (R)2)/4 X (1/∏(R)2) X 100) % = 75%