25.VOLUME AND SURFACE AREA

IMPORTANT FORMULAE

I. CUBOID

Let length = 1, breadth = b and height = h units. Then, 1. Volume = (1 x b x h) cubic units.

2. Surface area= 2(lb + bh + lh) sq.units.

3. Diagonal.=$\sqrt{l^2 + b^2 + h^2}$ units

II. CUBE

Let each edge of a cube be of length a. Then,

1. Volume = a^3 cubic units.

2. Surface area = $6a^2$ sq. units.

3. Diagonal = $\sqrt{3}$ a units.

III. CYLINDER

Let radius of base = r and Height (or length) = h. Then,

1. Volume = ($\prod r^2 h$) cubic units.

2. Curved surface area = ($2\prod$ rh). units.

3. Total surface area =$2\prod r$ (h+r) sq. units

IV. CONE

Let radius of base = r and Height = h. Then,

1. Slant height, l =$\sqrt{h^2+r^2}$

2. Volume = (1/3) $\prod r^2 h$ cubic units.

3. Curved surface area = ($\prod$rl) sq. units.

4. Total surface area = ($\prod rl + \prod r^2$) sq. units.

V. SPHERE

Let the radius of the sphere be r. Then,

1. Volume = (4/3)$\prod$r3 cubic units.

2. Surface area = ($4\prod r^2$) sq. units.

VI. HEMISPHERE

Let the radius of a hemisphere be r. Then,

1. Volume = (2/3)$\prod r^3$ cubic units.

2. Curved surface area = ($2\prod r^2$) sq. units.

3. Total surface area = ($3\prod r^2$) units.

Remember: 1 litre = 1000 cm3.

SOLVED EXAMPLES

Ex. 1. Find the volume and surface area of a cuboid 16 m long, 14 m broad and 7 m high.

Sol. Volume = (16 x 14 x 7) m3 = 1568 m3.

1 Surface area = [2 (16 x 14 + 14 x 7 + 16 x 7)] cm^2 = (2 x 434) cm^2 = 868 cm^2.

Ex. 2. Find the length of the longest pole that can be placed in a room 12 m long 8m broad and 9m high.

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Sol. Length of longest pole = Length of the diagonal of the room

= $\sqrt{(12^2+8^2+9^2} = .\sqrt{(289)} = 17$ m.

Ex. 3. Tbe volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 cu. metres. Find the breadth of the wall.

Sol. Let the breadth of the wall be x metres.

Then, Height = 5x metres and Length = 40x metres.

$\therefore x * 5x * 40x = 12.8 \Leftrightarrow x^3=12.8/200 = 128/2000 = 64/1000$

So, x = (4/10) m =((4/10)*100)cm = 40 cm

Ex. 4. Find the number of bricks, each measuring 24 cm x 12 cm x 8 cm, required to construct a wall 24 m long, 8m high and 60 cm thick, if 10% of the wall is filled with mortar?

Sol. Volume of the wall = (2400 x 800 x 60) cu. cm.

Volume of bricks = 90% of the volume of the wall

=((90/100)*2400 *800 * 60)cu.cm.

Volume of 1 brick = (24 x 12 x 8) cu. cm.

$\therefore$Number of bricks=(90/100)*(2400*800*60)/(24*12*8)=45000.

Ex. 5. Water flows into a tank 200 m x 160 m througb a rectangular pipe of 1.5m x 1.25 m @ 20 kmph . In what time (in minutes) will the water rise by 2 metres?

Sol. Volume required in the tank = (200 x 150 x 2) m^3 = 60000 m^3.

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Length of water column flown in1 min =(20*1000)/60 m =1000/3 m

Volume flown per minute = 1.5 * 1.25 * (1000/3) m^3 = 625 m^3.

$\therefore$ Required time = (60000/625)min = 96min

Ex. 6. Tbe dimensions of an open box are 50 cm, 40 cm and 23 cm. Its thickness is 2 cm. If 1 cubic cm of metal used in the box weighs 0.5 gms, find the weight of the box.

Sol. Volume of the metal used in the box = External Volume - Internal Volume

= [(50 * 40 * 23) - (44 * 34 * 20)]cm^3

= 16080 cm^3

$\therefore$ Weight of the metal =((16080*0.5)/1000) kg = 8.04 kg.

Ex. 7. The diagonal of a cube is $6\sqrt{3}$cm. Find its volume and surface area.

Sol. Let the edge of the cube be a.

$\therefore \sqrt{3}a = 6../3$ _ a = 6.

So,Volume = a^3 = (6 x 6 x 6) cm^3 = 216 cm^3.

Surface area = $6a^2$ = (6 x 6 x 6) cm^2 == 216 cm^2.

Ex. 8. The surface area of a cube is 1734 sq. cm. Find its volume.

Sol. Let the edge of the cube bea. Then,

$6a^2 = 1734 \Rightarrow a^2 = 289$ => a = 17 cm.

$\therefore$ Volume = $a^3 = (17)^3$ cm^3 = 4913 cm^3.

Ex. 9. A rectangular block 6 cm by 12 cm by 15 cm is cut up into an exact number of equal cubes. Find the least possible number of cubes.

Sol. Volume of the block = (6 x 12 x 15) cm^3 = 1080 cm^3.

Side of the largest cube = H.C.F. of 6 cm, 12 cm, 15 cm = 3 cm.

Volume of this cube = (3 x 3 x 3) cm^3 = 27 cm^3.

Number of cubes = 1080/27 = 40.

Ex.l0. A cube of edge 15 cm is immersed completely in a rectangular vessel containing water . If the dimensions of the base of vessel are 20 cm x 15 cm, find the rise in water level.

Sol. Increase in volume = Volume of the cube = (15 x 15 x 15) cm^3.

$\therefore$Rise in water level = volume/area = (15 x 15 x 15)/(20 x 15) cm = 11.25 cm.

Ex. 11. Three solid cubes of sides 1 cm, 6 cm and 8 cm are melted to form a new cube. Find the surface area of the cube so formed.

Sol. Volume of new cube = $(1^3 + 6^3 + 8^3)$ cm+ = 729 cm^3.

Edge of new cube = $_3\sqrt{729}$ cm = 9 cm.

$\therefore$ Surface area of the new cube = (6 x 9 x 9) cm^2 = 486 cm^2.

Ex. 12. If each edge of a cube is increased by 50%, find the percentage increase in **Its surface area.** "

Sol. Let original length of each edge = a.

Then, original surface area = $6a^2$.

New edge = (150% of a) = (150a/100) = 3a/2

New surface area = $6x(3a/2)^2 = 27a^2/2$

Increase percent in surface area = $\left(\left(\frac{15a^2}{2}\right) \times \left(\frac{1}{6a^2}\right) \times 100\right)\% = 125\%$

Ex. 13. Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface areas.

Sol. Let their edges be a and b. Then,

$a^3/b^3 = 1/27$ (or) $(a/b)^3 = (1/3)^3$ (or) $(a/b) = (1/3)$.

$\therefore$ Ratio of their surface area = $6a^2/6b^2 = a^2/b^2 = (a/b)^2 = 1/9$, i.e. 1:9.

Ex.14.Find the volume , curved surface area and the total surface area of a cylinder with diameter of base 7 cm and height 40 cm.

Sol. Volume = $\prod r^2 h = ((22/7)x(7/2)x(7/2)x40) = 1540\ cm^3$.

Curved surface area = $2\prod rh = (2x(22/7)x(7/2)x40) = 880\ cm^2$.

Total surface area = $2\prod rh + 2\prod r^2 = 2\prod r\,(h + r)$

$= (2 \times (22/7) \times (7/2) \times (40+3.5))\ cm^2$

$= 957\ cm^2$

Ex.15. If the capacity of a cylindrical tank is 1848 m^3 and the diameter of its base is 14 m, then find the depth of the tank.

Sol. Let the depth of the tank be h metres. Then,

$\prod \times 7^2 \times h = 1848 \Rightarrow h = (1848 \times (7/22) \times (1/49)) = 12$ m

Ex.16. 2.2 cubic dm of lead is to be drawn into a cylindrical wire 0.50 cm **diameter. Find the length of the wire in metres.**

Sol. Let the length of the wire be h metres. Then,

$\prod\ (0.50/(2 \times 100))^2 \times h = 2.2/1000$

$\Rightarrow h = ((2.2/1000) \times (100 \times 100)/(0.25 \times 0.25) \times (7/22)) = 112$ m.

Ex. 17. How many iron rods, each of length 7 m and diameter 2 cm can be made out of 0.88 cubic metre of iron?

Sol. Volume of 1 rod = ((22/7) x (1/100) x (1/100) x 7) cu.m = 11/5000 cu.m

Volume of iron = 0.88 cu. m.

Number of rods = (0.88 x 5000/11) = 400.

Ex. 18. The radii of two cylinders are in the ratio 3: 5 and their heights are in tbe ratio of 2 : 3. Find the ratio of their curved surface areas.

Sol. Let the radii of the cylinders be 3x, 5x and their heights be 2y, 3y respectively. Then

Ratio of their curved surface area = $\frac{2\prod X 3x X 2y}{2\prod X 5x X 3y}$ = 2/5 = 2.5

Ex. 19. If 1 cubic cm of cast iron weighs 21 gms, then find the eight of a cast iron pipe of length 1 metre with a bore of 3 cm and in which thickness of the metal is 1 em.

Sol. Inner radius = (3/2) cm = 1.5 cm, Outer radius = (1.5 + 1) = 2.5 cm.

$\therefore$ Volume of iron = $[\prod \text{ x } (2.5)^2 \text{ x } 100 - \prod \text{ x } (1.5)^2 \text{ x } 100] \text{ cm}^3$

$= (22/7) \text{ x } 100 \text{ x } [(2.5)^2 - (1.5)^2] \text{ cm}^3$

$= (8800/7) \text{ cm}^3$

Weight of the pipe = ((8800/7) x (21/1000))kg = 26.4 kg.

Ex. 20. Find the slant height, volume, curved surface area and the whole surface area of a cone of radius 21 cm and height 28 cm.

Sol. Here, r = 21 cm and h = 28 cm.

$\therefore$ Slant height, l = $\sqrt{r^2 + h^2} = \sqrt{(21)^2 + (28)^2} = \sqrt{1225}$ = 35cm

Ex. 21. Find the length of canvas 1.25 m wide required to build a conical tent of base radius 7 metres and height 24 metres.

Sol. Here, r = 7m and h = 24 m.

So,l = $\sqrt{(r^2 + h^2)} = \sqrt{(7^2 + 24^2)} = \sqrt{(625)}$ = 25 m.
Area of canvas = $\prod$rl=((22/7)*7*25)m^2 = 550 m^2 .
Length of canvas = (Area/Width) = (550/1.25) m = 440 m.

Ex. 22. The heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. Find the ratio of their volumes.

Sol. Let the radii of their bases be r and R and their heights be h and 2h respectively.

Then,$(2\prod r/2\prod R)=(3/4) \Rightarrow R=(4/3)r$.

$\therefore$ Ratio of volumes = $(((1/3)\prod r^2h)/((1/3)\prod(4/3r)^2(2h)))$=9 : 32.

Ex. 23. The radii of the bases of a cylinder and a cone are in the ratio of 3 : 4 and It heights are in the ratio 2 : 3. Find the ratio of their volumes.

Sol. Let the radii of the cylinder and the cone be 3r and 4r and their heights be 2h and 3h respectively.

$\therefore \frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{\prod \text{ x } (3r)^2 * 2h}{(1/3)\prod r^2 * 3h}$ = 9/8 = 9 : 8.

Ex. 24. A conical vessel, whose internal radius is 12 cm and height 50 cm, is full of

liquid. The contents are emptied into a cylindrical vessel with internal radius 10 cm. Find the height to which the liquid rises in the cylindrical vessel.

Sol. Volume of the liquid in the cylindrical vessel
= Volume of the conical vessel
= ((1/3)* (22/7)* 12 * 12 * 50))cm^3 = (22 *4 *12 * 50)/7 cm^3.

Let the height of the liquid in the vessel be h.

Then (22/7)*10*10*h =(22*4*12*50)/7 or h = (4*12*50)/100 = 24 cm

Ex. 25. Find the volume and surface area of a sphere of radius 10.5 cm.

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Sol. Volume = (4/3)∏r^3 =(4/3)*(22/7)*(21/2)*(21/2)*(21/2) cm^3 = 4851 cm^3.
Surface area = 4∏r^2 =(4*(22/7)*(21/2)*(21/2)) cm^2 = 1386 cm^2

Ex. 26. If the radius of a sphere is increased by 50%, find the increase percent in volume and the increase percent in the surface area.

Sol. Let original radius = R. Then, new radius = (150/100)R=(3R/2)
Original volume = (4/3)∏R^3, New volume = (4/3)∏$(3R/2)^3$ =(9∏R^3/2)
Increase % in volume=((19/6)∏R^3)*(3/4∏R^3)*100))% = 237.5%
Original surface area =4∏R^2. New surface area = 4∏$(3R/2)^2$=9∏R^2
Increase % in surface area =(5∏R^2/4∏R^2) * 100) % = 125%.

Ex. 27. Find the number of lead balls, each 1 cm in diameter that can be a sphere of diameter 12 cm.

Sol. Volume of larger sphere = (4/3)∏*6*6*6) cm^3 = 288∏ cm^3.
Volume of 1 small lead ball = ((4/3)∏*(1/2)*(1/2)*(1/2)) cm^3 = ∏/6 cm^3.
∴ Number of lead balls = (288∏*(6/∏)) = 1728.

Ex.28.How many spherical bullets can be made out of a lead cylinder 28cm high and with radius 6 cm, each bullet being 1.5 cm in diameter ?

Sol. Volume of cylinder = (∏ x 6 x 6 x 28) cm^3 = (9∏/16) cm^3.
Number of bullet = $\dfrac{\text{Volume of cylinder}}{\text{Volume of each bullet}}$ = [(36 x 28)∏ x 16] /9∏ = 1792.

Ex.29.A copper sphere of diameter 18cm is drawn into a wire of diameter 4 mm Find the length of the wire.

Sol. Volume of sphere = ((4∏/3) x 9 x 9 x 9) cm^3 = 972∏ cm^3
Volume of sphere = (∏ x 0.2 x 0.2 x h) cm^3
∴ 972∏ = ∏ x (2/10) x (2/10) x h ⇒ h = (972 x 5 x 5)cm = [(972 x 5 x5)/100] m
= 243m

Ex.30.Two metallic right circular cones having their heights 4.1 cm and 4.3 cm and the radii of their bases 2.1 cm each, have been melted together and recast into a

sphere. Find the diameter of the sphere.

Sol. Volume of sphere = Volume of 2 cones

$$= \left(\frac{1}{3}\prod \times (2.10^2) \times 4.1 + \frac{1}{3}\prod \times (2.1)^2 \times 4.3\right)$$

Let the radius of sphere be R

$\therefore (4/3)\prod R^3 = (1/3)\prod(2.1)^3$ or $R = 2.1$cm

Hence , diameter of the sphere = 4.2.cm

Ex.31.A Cone and a sphere have equal radii and equal volumes. Find the ratio of the sphere of the diameter of the sphere to the height of the cone.

Sol. Let radius of each be R and height of the cone be H.

Then, $(4/3)\prod R^3 = (1/3)\prod R^2H$ (or) $R/H = ¼$ (or) $2R/H = 2/4 = 1/2$

$\therefore$Required ratio = 1:2.

Ex.32.Find the volume , curved surface area and the total surface area of a hemisphere of radius 10.5 cm.

Sol. Volume = $(2\prod r^3/3) = ((2/3) \times (22/7) \times (21/2) \times (21/2) \times (21/2))\text{cm}^3$

$= 2425.5 \text{ cm}^3$

Curved surface area = $2\prod r^3 = (2 \times (22/7) \times (21/2) \times (21/2))\text{cm}^2$

$=693 \text{ cm}^2$

Total surface area = $3\prod r^3 = (3 \times (22/7) \times (21/2) \times (21/2))\text{cm}^2$

$= 1039.5 \text{ cm}^2$.

Ex.33.Hemispherical bowl of internal radius 9 cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3 cm and height 4 cm. How many bottles will be needed to empty the bowl ?

Sol. Volume of bowl = $((2\prod/3) \times 9 \times 9 \times 9)\text{ cm}^3 = 486\prod \text{ cm}^3$.

Volume of 1 bottle = $(\prod \times (3/2) \times (3/2) \times 4)\text{ cm}^3 = 9\prod \text{ cm}^3$

Number of bottles = $(486\prod/9\prod) = 54$.

Ex34.A Cone,a hemisphere and a cylinder stand on equal bases and have the same height.Find ratio of their volumes.

Sol. Let R be the radius of each

Height of the hemisphere = Its radius = R.

$\therefore$Height of each = R.

Ratio of volumes = $(1/3)\prod R^2 \times R : (2/3)\prod R^3 : \prod R^2 \times R = 1:2:3$
