

3. DECIMAL FRACTIONS

IMPORTANT FACTS AND FORMULAE

I. Decimal Fractions : Fractions in which denominators are powers of 10 are known as **decimal fractions**.

Thus, $1/10 = 1 \text{ tenth} = .1$; $1/100 = 1 \text{ hundredth} = .01$;

$99/100 = 99 \text{ hundredths} = .99$; $7/1000 = 7 \text{ thousandths} = .007$, etc

II. Conversion of a Decimal Into Vulgar Fraction : Put 1 in the denominator under the decimal point and annex with it as many zeros as is the number of digits after the decimal point. Now, remove the decimal point and reduce the fraction to its lowest terms.

Thus, $0.25 = 25/100 = 1/4$; $2.008 = 2008/1000 = 251/125$.

III. 1. Annexing zeros to the extreme right of a decimal fraction does not change its value
Thus, $0.8 = 0.80 = 0.800$, etc.

2. If numerator and denominator of a fraction contain the same number of decimal places, then we remove the decimal sign.
Thus, $1.84/2.99 = 184/299 = 8/13$; $0.365/0.584 = 365/584 = 5$

IV. Operations on Decimal Fractions :

1. Addition and Subtraction of Decimal Fractions : The given numbers are so placed under each other that the decimal points lie in one column. The numbers so arranged can now be added or subtracted in the usual way.

2. Multiplication of a Decimal Fraction By a Power of 10 : Shift the decimal point to the right by as many places as is the power of 10.

Thus, $5.9632 \times 100 = 596.32$; $0.073 \times 10000 = 0.0730 \times 10000 = 730$.

3. Multiplication of Decimal Fractions : Multiply the given numbers considering them without the decimal point. Now, in the product, the decimal point is marked off to obtain as many places of decimal as is the sum of the number of decimal places in the given numbers.

Suppose we have to find the product $(.2 \times .02 \times .002)$. Now, $2 \times 2 \times 2 = 8$. Sum of decimal places $= (1 + 2 + 3) = 6$. $.2 \times .02 \times .002 = .000008$.

4. Dividing a Decimal Fraction By a Counting Number : Divide the given number without considering the decimal point, by the given counting number. Now, in the quotient, put the decimal point to give as many places of decimal as there are in the dividend.

Suppose we have to find the quotient $(0.0204 \div 17)$. Now, $204 \div 17 = 12$. Dividend contains 4 places of decimal. So, $0.0204 \div 17 = 0.0012$.

5. Dividing a Decimal Fraction By a Decimal Fraction : Multiply both the dividend and the divisor by a suitable power of 10 to make divisor a whole number. Now, proceed as above.

Thus, $0.00066/0.11 = (0.00066*100)/(0.11*100) = (0.066/11) = 0.006V$

V. Comparison of Fractions : Suppose some fractions are to be arranged in ascending or descending order of magnitude. Then, convert each one of the given fractions in the decimal form, and arrange them accordingly.

Suppose, we have to arrange the fractions $3/5$, $6/7$ and $7/9$ in descending order.

now, $3/5=0.6$, $6/7 = 0.857$, $7/9 = 0.777....$

since $0.857>0.777...>0.6$, so $6/7>7/9>3/5$

VI. Recurring Decimal : If in a decimal fraction, a figure or a set of figures is repeated continuously, then such a number is called a *recurring decimal*.

In a recurring decimal, if a single figure is repeated, then it is expressed by putting a dot on it. If a set of figures is repeated, it is expressed by putting a bar on the set

Thus $1/3 = 0.3333.... = 0.\dot{3}$; $22/7 = 3.142857142857..... = 3.\overline{142857}$

Pure Recurring Decimal: A decimal fraction in which all the figures after the decimal point are repeated, is called a pure recurring decimal.

Converting a Pure Recurring Decimal Into Vulgar Fraction : Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures.

thus $.5 = 5/9$; $0.53 = 53/99$; $0.067 = 67/999$; etc...

Mixed Recurring Decimal: A decimal fraction in which some figures do not repeat and some of them are repeated, is called a mixed recurring decimal.

e.g., $0.17333... = 0.17\dot{3}$.

Converting a Mixed Recurring Decimal Into Vulgar Fraction : In the numerator, take the difference between the number formed by all the digits after decimal point (taking repeated digits only once) and that formed by the digits which are not repeated, In the denominator, take the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

Thus $0.16 = (16-1) / 90 = 15/90 = 1/6$;

$0.\overline{2273} = (2273 - 22)/9900 = 2251/9900$

VII. Some Basic Formulae :

1. $(a + b)(a - b) = (a^2 - b^2)$.
2. $(a + b)^2 = (a^2 + b^2 + 2ab)$.
3. $(a - b)^2 = (a^2 + b^2 - 2ab)$.
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
5. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
6. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$.
7. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
8. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

SOLVED EXAMPLES

Ex. 1. Convert the following into vulgar fraction:

(i) 0.75 (ii) 3.004 (iii) 0.0056

Sol. (i). $0.75 = 75/100 = 3/4$ (ii) $3.004 = 3004/1000 = 751/250$ (iii) $0.0056 = 56/10000 = 7/1250$

Ex. 2. Arrange the fractions $5/8$, $7/12$, $13/16$, $16/29$ and $3/4$ in ascending order of magnitude.

Sol. Converting each of the given fractions into decimal form, we get :

$5/8 = 0.625$, $7/12 = 0.5833$, $16/29 = 0.5517$, and $3/4 = 0.75$

Now, $0.5517 < 0.5833 < 0.625 < 0.75 < 0.8125$

$\therefore 16/29 < 7/12 < 5/8 < 3/4 < 13/16$

Ex. 3. arrange the fractions $3/5$, $4/7$, $8/9$, and $9/11$ in their descending order.

Sol. Clearly, $3/5 = 0.6$, $4/7 = 0.571$, $8/9 = 0.88$, $9/11 = 0.818$.

Now, $0.88 > 0.818 > 0.6 > 0.571$

$\therefore 8/9 > 9/11 > 3/5 > 4/7$

Ex. 4. Evaluate : (i) $6202.5 + 620.25 + 62.025 + 6.2025 + 0.62025$

(ii) $5.064 + 3.98 + 0.7036 + 7.6 + 0.3 + 2$

Sol. (i) 6202.5

620.25

62.025

6.2025

+ 0.62025

6891.59775

(ii) 5.064

3.98

0.7036

7.6

0.3

2.0

19.6476

Ex. 5. Evaluate : (i) $31.004 - 17.2368$

(ii) $13 - 5.1967$

Sol. (i)
$$\begin{array}{r} 31.0040 \\ - 17.2386 \\ \hline 13.7654 \end{array}$$

(ii)
$$\begin{array}{r} 31.0000 \\ - 5.1967 \\ \hline 7.8033 \end{array}$$

Ex. 6. What value will replace the question mark in the following equations ?

(i) $5172.49 + 378.352 + ? = 9318.678$

(ii) $? - 7328.96 + 5169.38$

Sol. (i) Let $5172.49 + 378.352 + x = 9318.678$

Then, $x = 9318.678 - (5172.49 + 378.352) = 9318.678 - 5550.842 = 3767.836$

(ii) Let $x - 7328.96 = 5169.38$. Then, $x = 5169.38 + 7328.96 = 12498.34$.

Ex. 7. Find the products: (i) 6.3204×100

(ii) 0.069×10000

Sol. (i) $6.3204 \times 1000 = 632.04$

(ii) $0.069 \times 10000 = 0.0690 \times 10000 = 690$

Ex. 8. Find the product:

(i) 2.61×1.3

(ii) 2.1693×1.4

(iii) $0.4 \times 0.04 \times 0.004 \times 40$

Sol. (i) $261 \times 13 = 3393$. Sum of decimal places of given numbers $= (2+1) = 3$.

$2.61 \times 1.3 = 3.393$.

(ii) $21693 \times 14 = 303702$. Sum of decimal places $= (4+1) = 5$

$2.1693 \times 1.4 = 3.03702$.

(iii) $4 \times 4 \times 4 \times 40 = 2560$. Sum of decimal places $= (1 + 2 + 3) = 6$

$0.4 \times 0.04 \times 0.004 \times 40 = 0.002560$.

Ex. 9. Given that $268 \times 74 = 19832$, find the values of 2.68×0.74 .

Sol. Sum of decimal places $= (2 + 2) = 4$

$2.68 \times 0.74 = 1.9832$.

Ex. 10. Find the quotient:

(i) $0.63 \div 9$

(ii) $0.0204 \div 17$

(iii) $3.1603 \div 13$

Sol. (i) $63 \div 9 = 7$. Dividend contains 2 places decimal.

$0.63 \div 9 = 0.7$.

(ii) $204 \div 17 = 12$. Dividend contains 4 places of decimal.

$0.2040 \div 17 = 0.012$.

(iii) $31603 \div 13 = 2431$. Dividend contains 4 places of decimal.

$3.1603 \div 13 = 0.2431$.

Ex. 11. Evaluate :

(i) $35 + 0.07$

(ii) $2.5 + 0.0005$

(iii) $136.09 + 43.9$

Sol. (i) $35/0.07 = (35 \times 100) / (0.07 \times 100) = (3500 / 7) = 500$
(ii) $25/0.0005 = (25 \times 10000) / (0.0005 \times 10000) = 25000 / 5 = 5000$
(iii) $136.09/43.9 = (136.09 \times 10) / (43.9 \times 10) = 1360.9 / 439 = 3.1$

Ex. 12. What value will come in place of question mark in the following equation?

(i) $0.006 + ? = 0.6$ (ii) $? + 0.025 = 80$

Sol. (i) Let $0.006 / x = 0.6$, Then, $x = (0.006 / 0.6) = (0.006 \times 10) / (0.6 \times 10) = 0.06/6 = 0.01$
(ii) Let $x / 0.025 = 80$, Then, $x = 80 \times 0.025 = 2$

Ex. 13. If $(1 / 3.718) = 0.2689$, Then find the value of $(1 / 0.0003718)$.

Sol. $(1 / 0.0003718) = (10000 / 3.718) = 10000 \times (1 / 3.718) = 10000 \times 0.2689 = 2689.$

Ex. 14. Express as vulgar fractions : (i) $0.\overline{37}$ (ii) $0.\overline{053}$ (iii) $3.\overline{142857}$

Sol. (i) $0.\overline{37} = 37 / 99$. (ii) $0.\overline{053} = 53 / 999$

(iii) $3.\overline{142857} = 3 + 0.\overline{142857} = 3 + (142857 / 999999) = 3 (142857/999999)$

Ex. 15. Express as vulgar fractions : (i) $0.1\overline{7}$ (ii) $0.12\overline{54}$ (iii) $2.5\overline{36}$

Sol. (i) $0.1\overline{7} = (17 - 1)/90 = 16 / 90 = 8/45$

(ii) $0.12\overline{54} = (1254 - 12) / 9900 = 1242 / 9900 = 69 / 550$

(iii) $2.5\overline{36} = 2 + 0.5\overline{36} = 2 + (536 - 53)/900 = 2 + (483/900) = 2 + (161/300) = 2 (161/300)$

Ex. 16. Simplify: $\frac{0.05 \times 0.05 \times 0.05 + 0.04 \times 0.04 \times 0.04}{0.05 \times 0.05 - 0.05 \times 0.04 + 0.04 \times 0.04}$

Sol. Given expression $= (a^3 + b^3) / (a^2 - ab + b^2)$, where $a = 0.05$, $b = 0.04$
 $= (a + b) = (0.05 + 0.04) = 0.09$