30. PERMUTATIONS AND COMBINATIONS

IMPORTANT FACTS AND FORMULAE

Factorial Notation: Let n be a positive integer. Then, factorial n, denoted by n! is defined as:

$$n! = n(n-1)(n-2)........3.2.1.$$

Examples: (i) 5! = (5x 4 x 3 x 2 x 1) = 120; (ii) 4! = (4x3x2x1) = 24 etc.
We define, 0! = 1.

Permutations: The different arrangements of a given number of things by taking some or all at a time, are called permutations.

Ex. 1. All permutations (or arrangements) made with the letters a, b, c by taking two at a time are: **(ab, ba, ac, bc, cb).**

Ex. 2. All permutations made with the letters a,b,c, taking all at a time are:
(abc, acb, bca, cab, cba).

Number of Permutations: Number of all permutations of n things, taken r at a time, given by:

$$^nP_r = n(n-1)(n-2).....(n-r+1) = n!/(n-r)!$$

Examples: (i) $^6p_2$ = (6x5) = 30. (ii) $^7p_3$ = (7x6x5) = 210.

Cor. Number of all permutations of n things, taken all at a time = n!

An Important Result: If there are n objects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of rth kind, such that $(p_1+p_2+.......p_r) = n$.

Then, number of permutations of these n objects is:

$$n! \;/\; (p_1!).p_2!)......(p_r!)$$

Combinations: Each of the different groups or selections which can be formed by taking some or all of a number of objects, is called a combination.

Ex. 1. Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.
Note that AB and BA represent the same selection.

Ex. 2. All the combinations formed by a, b, c, taking two at a time are **ab, bc, ca.**

Ex. 3. The only combination that can be formed of three letters a, b, c taken all at a time is **abc.**

Ex. 4. Various groups of 2 out of four presons A, B, C, D are:

AB, AC, AD, BC, BD, CD.

Ex. 5. Note that ab and ba are two different permutations but they represent the same combination.

Number of Combinations: The number of all combination of n things, taken r at a time is:

$$^{n}C_{r} = n! / (r!)(n-r)! = n(n-1)(n-2).....\text{to } r \text{ factors} / r!$$

Note that: $^{n}c_{r} = 1$ and $^{n}c_{0} = 1$.

An Important Result: $^{n}c_{r} = {}^{n}c_{(n-r)}$.

Example: (i) $^{11}c_{4} = (11x10x9x8)/(4x3x2x1) = 330$.

(ii) $^{16}c_{13} = {}^{16}c_{(16-13)} = 16x15x14/3! = 16x15x14/3x2x1 = 560$.

SOLVED EXAMPLES

Ex. 1. Evaluate: 30!/28!

Sol. We have, 30!/28! = 30x29x(28!)/28! = (30x29) = 870.

Ex. 2. Find the value of (i) $^{60}p_{3}$ (ii) $^{4}p_{4}$

Sol. (i) $^{60}p_{3} = 60!/(60-3)! = 60!/57! = 60x59x58x(57!)/57! = (60x59x58) = 205320$.

(ii) $^{4}p_{4} = 4! = (4x3x2x1) = 24$.

Ex. 3. Find the vale of (i) $^{10}c_{3}$ (ii) $^{100}c_{98}$ (iii) $^{50}c_{50}$

Sol. (i) $^{10}c_{3} = 10x9x8/3! = 120$.

(ii) $^{100}c_{98} = {}^{100}c_{(100-98)} = 100x99/2! = 4950$.

(iii) $^{50}c_{50} = 1$. $[^{n}c_{n} = 1]$

Ex. 4. How many words can be formed by using all letters of the word "BIHAR"

Sol. The word BIHAR contains 5 different letters.

Required number of words = $^{5}p_{5}$ = 5! = (5x4x3x2x1) = 120.

Ex. 5. How many words can be formed by using all letters of the word 'DAUGHTER' so that the vowels always come together?

Sol. Given word contains 8 different letters. When the vowels AUE are always together, we may suppose them to form an entity, treated as one letter.
Then, the letters to be arranged are DGNTR (AUE).

Then 6 letters to be arranged in $^{6}p_{6}$ = 6! = 720 ways.

The vowels in the group (AUE) may be arranged in 3! = 6 ways.

Required number of words = (720x6) = 4320.

Ex. 6. How many words can be formed from the letters of the word 'EXTRA' so that the vowels are never together?

Sol. The given word contains 5 different letters.
Taking the vowels EA together, we treat them as one letter.
Then, the letters to be arranged are XTR (EA).
These letters can be arranged in 4! = 24 ways.
The vowels EA may be arranged amongst themselves in 2! = 2 ways.
Number of words, each having vowels together = (24x2) = 48 ways.
Total number of words formed by using all the letters of the given words
= 5! = (5x4x3x2x1) = 120.

Number of words, each having vowels never together = (120-48) = 72.

Ex. 7. How many words can be formed from the letters of the word 'DIRECTOR'
So that the vowels are always together?

Sol. In the given word, we treat the vowels IEO as one letter.
Thus, we have DRCTR (IEO).
This group has 6 letters of which R occurs 2 times and others are different.
Number of ways of arranging these letters = 6!/2! = 360.
Now 3 vowels can be arranged among themselves in 3! = 6 ways.
Required number of ways = (360x6) = 2160.

Ex. 8. In how many ways can a cricket eleven be chosen out of a batch of 15 players ?

Sol. Required number of ways = ${}^{15}c_{11} = {}^{15}c_{(15-11)} = {}^{11}c_4$

$= 15x14x13x12/4x3x2x1 = 1365.$

Ex. 9. In how many ways, a committee of 5 members can be selected from 6 men and 5 ladies, consisting of 3 men and 2 ladies?

Sol. (3 men out 6) and (2 ladies out of 5) are to be chosen.

Required number of ways = $({}^6c_3 x {}^5c_2) = [6x5x4/3x2x1] \text{ x } [5x4/2x1] = 200.$