

31. PROBABILITY

IMPORTANT FACTS AND FORMULA

1.Experiment :An operation which can produce some well-defined outcome is called an experiment

2.Random experiment: An experiment in which all possible outcome are known and the exact out put cannot be predicted in advance is called an random experiment

Eg of performing random experiment:

- (i)rolling an unbiased dice
- (ii)tossing a fair coin
- (iii)drawing a card from a pack of well shuffled card
- (iv)picking up a ball of certain color from a bag containing ball of different colors

Details:

- (i)when we throw a coin. Then either a **head(h)** or a **tail (t)** appears.
- (ii)a dice is a solid cube, having 6 faces ,marked 1,2,3,4,5,6 respectively when we throw a die , the outcome is the number that appear on its top face .
- (iii)a pack of cards has 52 cards it has 13 cards of each suit ,namely spades, clubs ,hearts and diamonds
 - Cards of spades and clubs are black cards
 - Cards of hearts and diamonds are red cards
 - There are 4 honors of each suit
 - These are **aces ,king ,queen and jack**
 - These are called face cards

3.Sample space :When we perform an experiment ,then the set S of all possible outcome is called the sample space

eg of sample space:

- (i)in tossing a coin , $s=\{h,t\}$
- (ii)if two coin are tossed ,then $s=\{hh,tt,ht,th\}$.
- (iii)in rolling a die we have, $s=\{1,2,3,4,5,6\}$.

4.event:Any subset of a sample space.

5.Probability of occurrence of an event.

let S be the sample space and E be the event .

then, $E \subseteq S$.

$P(E)=n(E)/n(S)$.

6.Results on Probability:

- (i) $P(S) = 1$ (ii) $0 \leq P(E) \leq 1$ (iii) $P(\phi)=0$
- (iv)For any event a and b, we have:
 $P(a \cup b)=P(a)+P(b)-P(a \cap b)$

(v) If \bar{A} denotes (not-a), then $P(\bar{A}) = 1 - P(A)$.

SOLVED EXAMPLES

Ex 1. In a throw of a coin, find the probability of getting a head.

sol. Here $S = \{H, T\}$ and $E = \{H\}$.

$$P(E) = n(E)/n(S) = 1/2$$

Ex2. Two unbiased coin are tossed. what is the probability of getting atmost one head?

sol. Here $S = \{HH, HT, TH, TT\}$

Let E = event of getting one head

$$E = \{TT, HT, TH\}$$

$$P(E) = n(E)/n(S) = 3/4$$

Ex3. An unbiased die is tossed. find the probability of getting a multiple of 3

sol. Here $S = \{1, 2, 3, 4, 5, 6\}$

Let E be the event of getting the multiple of 3

$$\text{then } E = \{3, 6\}$$

$$P(E) = n(E)/n(S) = 2/6 = 1/3$$

Ex4. In a simultaneous throw of pair of dice. find the probability of getting the total more than 7

sol. Here $n(S) = (6 \times 6) = 36$

let E = event of getting a total more than 7

$$= \{(2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

$$P(E) = n(E)/n(S) = 15/36 = 5/12.$$

Ex5. A bag contains 6 white and 4 black balls. 2 balls are drawn at random. find the probability that they are of same colour.

Sol. let S be the sample space

$$\text{Then } n(S) = \text{no of ways of drawing 2 balls out of } (6+4) = {}^{10}C_2 = (10 \times 9)/(2 \times 1) = 45$$

Let E = event of getting both balls of same colour

$$\text{Then } n(E) = \text{no of ways (2 balls out of six) or (2 balls out of 4)}$$

$$= ({}^6C_2 + {}^4C_2) = (6 \times 5)/(2 \times 1) + (4 \times 3)/(2 \times 1) = 15 + 6 = 21$$

$$P(E) = n(E)/n(S) = 21/45 = 7/15$$

Ex6. Two dice are thrown together. What is the probability that the sum of the number on the two faces is divided by 4 or 6

sol. Clearly $n(S) = 6 \times 6 = 36$

Let E be the event that the sum of the numbers on the two faces is divided by 4 or 6. Then

$E = \{(1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (5,1), (5,3), (6,2), (6,6)\}$

$n(E) = 14$.

Hence $p(e) = n(e)/n(s) = 14/36 = 7/18$

Ex7. Two cards are drawn at random from a pack of 52 cards. what is the probability that either both are black or both are queen?

sol. We have $n(s) = {}^{52}C_2 = (52 \cdot 51)/(2 \cdot 1) = 1326$.

Let A = event of getting both black cards

B = event of getting both queens

$A \cap B$ = event of getting queen of black cards

$n(A) = {}^{26}C_2 = (26 \cdot 25)/(2 \cdot 1) = 325$,

$n(B) = {}^4C_2 = (4 \cdot 3)/(2 \cdot 1) = 6$ and

$n(A \cap B) = {}^2C_2 = 1$

$P(A) = n(A)/n(S) = 325/1326$;

$P(B) = n(B)/n(S) = 6/1326$ and

$P(A \cap B) = n(A \cap B)/n(S) = 1/1326$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = (325 + 6 - 1)/1326 = 330/1326 = 55/221$