

## 34. HEIGHTS AND DISTANCES

### IMPORTANT FACTS AND FORMULAE

1. We already know that:

In a rt.angled  $\Delta OAB$ , where  $\angle BOA = \theta$ ,

i)  $\sin \theta = \text{Perpendicular/Hypotenuse} = AB/OB$ ;

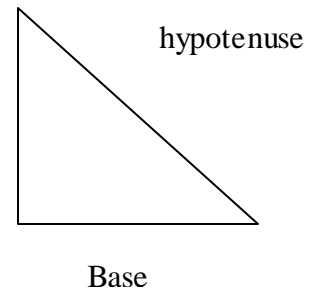
ii)  $\cos \theta = \text{Base/Hypotenuse} = OA/OB$ ;

iii)  $\tan \theta = \text{Perpendicular/Base} = AB/OA$ ;

iv)  $\text{cosec } \theta = 1/\sin \theta = OB/AB$ ;

v)  $\sec \theta = 1/\cos \theta = OB/OA$ ;

vi)  $\cot \theta = 1/\tan \theta = OA/AB$ .



2. **Trigonometrical identities:**

i)  $\sin^2 \theta + \cos^2 \theta = 1$ .

ii)  $1 + \tan^2 \theta = \sec^2 \theta$

iii)  $1 + \cot^2 \theta = \text{cosec}^2 \theta$

3. **Values of T-ratios:-**

$\theta$	0	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined

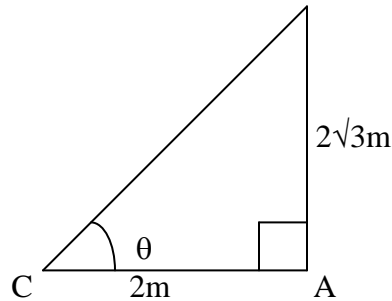
4. **Angle of Elevation:** Suppose a man from a point O looks up an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of elevation of P as seen from O.

$\therefore$  Angle of elevation of P from O =  $\angle AOP$ .

5. **Angle of Depression:** Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the angle of depression of P as seen from O.

## SOLVED EXAMPLES

**Ex.1.** If the height of a pole is  $2\sqrt{3}$  metres and the length of its shadow is 2 metres, find the angle of elevation of the sun.

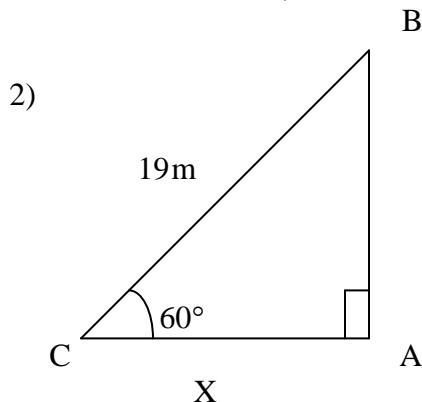


**Sol.** Let AB be the pole and AC be its shadow.  
Let angle of elevation,  $\angle ACB = \theta$ .  
Then,  $AB = 2\sqrt{3}$  m  $AC = 2$  m.

$$\tan \theta = AB/AC = 2\sqrt{3}/2 = \sqrt{3} \Rightarrow \theta = 60^\circ$$

So, the angle of elevation is  $60^\circ$

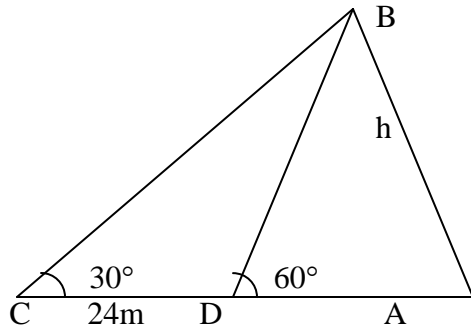
**Ex.2.** A ladder leaning against a wall makes an angle of  $60^\circ$  with the ground. If the length of the ladder is 19 m, find the distance of the foot of the ladder from the wall.



**Sol.** Let AB be the wall and BC be the ladder.  
Then,  $\angle ACB = 60^\circ$  and  $BC = 19$  m.  
Let  $AC = x$  metres  
 $AC/BC = \cos 60^\circ \Rightarrow x/19 = 1/2 \Rightarrow x = 19/2 = 9.5$

$\therefore$  Distance of the foot of the ladder from the wall = 9.5 m

**Ex.3. The angle of elevation of the top of a tower at a point on the ground is  $30^\circ$ . On walking 24 m towards the tower, the angle of elevation becomes  $60^\circ$ . Find the height of the tower.**



**Sol.** Let AB be the tower and C and D be the points of observation. Then,

$$AB/AD = \tan 60^\circ = \sqrt{3} \quad \Rightarrow \quad AD = AB/\sqrt{3} = h/\sqrt{3}$$

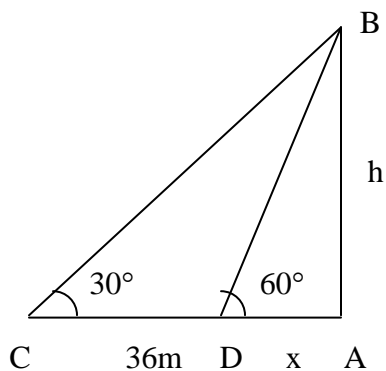
$$AB/AC = \tan 30^\circ = 1/\sqrt{3} \quad \Rightarrow \quad AC = AB \times \sqrt{3} = h\sqrt{3}$$

$$CD = (AC - AD) = (h\sqrt{3} - h/\sqrt{3})$$

$$h\sqrt{3} - h/\sqrt{3} = 24 \quad \Rightarrow \quad h = 12\sqrt{3} = (12 \times 1.73) = 20.76$$

Hence, the height of the tower is 20.76 m.

**Ex.4. A man standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is  $60^\circ$ . When he retires 36 m from the bank, he finds the angle to be  $30^\circ$ . Find the breadth of the river.**



Sol. Let AB be the tree and AC be the river. Let C and D be the two positions of the man. Then,

$\angle ACB = 60^\circ$ ,  $\angle ADB = 30^\circ$  and  $CD = 36$  m.

Let  $AB = h$  metres and  $AC = x$  metres.

Then,  $AD = (36 + x)$  metres.

$$\frac{AB}{AD} = \tan 30^\circ = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \frac{h}{(36+x)} = \frac{1}{\sqrt{3}} \quad \dots(1)$$

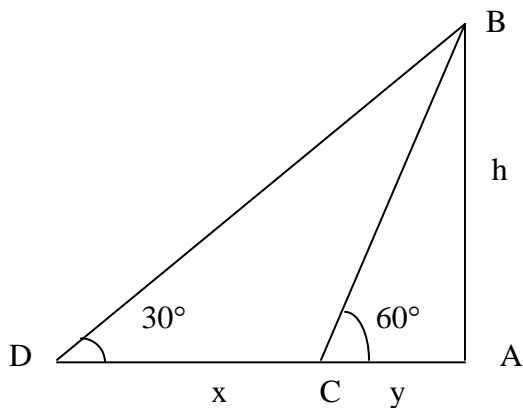
$$\frac{AB}{AC} = \tan 60^\circ = \sqrt{3} \quad \Rightarrow \quad \frac{h}{x} = \sqrt{3} \quad \dots(2)$$

From (i) and (ii), we get:

$$\frac{(36+x)}{\sqrt{3}} = \sqrt{3}x \quad \Rightarrow \quad x = 18 \text{ m.}$$

So, the breadth of the river = 18 m.

**Ex.5. A man on the top of a tower, standing on the seashore finds that a boat coming towards him takes 10 minutes for the angle of depression to change from  $30^\circ$  to  $60^\circ$ . Find the time taken by the boat to reach the shore from this position.**



Sol. Let AB be the tower and C and D be the two positions of the boat.

Let  $AB = h$ ,  $CD = x$  and  $AD = y$ .

$$\frac{h}{y} = \tan 60^\circ = \sqrt{3} \quad \Rightarrow \quad y = \frac{h}{\sqrt{3}}$$

$$\frac{h}{(x+y)} = \tan 30^\circ = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad x+y = \sqrt{3}h$$

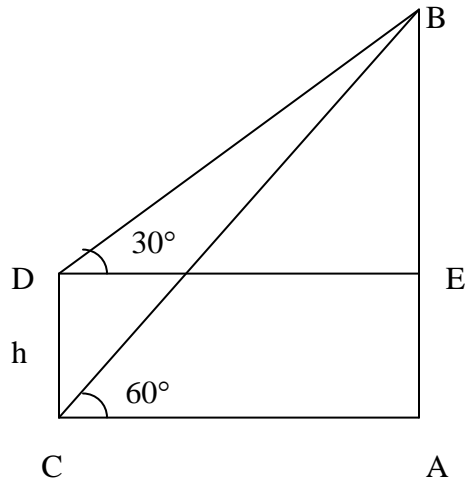
$$x = (x+y) - y = (\sqrt{3}h - \frac{h}{\sqrt{3}}) = \frac{2h}{\sqrt{3}}$$

Now,  $\frac{2h}{\sqrt{3}}$  is covered in 10 min.

$$\frac{h}{\sqrt{3}} \text{ will be covered in } (10 \times (\frac{\sqrt{3}}{2h}) \times (\frac{h}{\sqrt{3}})) = 5 \text{ min}$$

Hence, required time = 5 minutes.

**Ex 6.** There are two temples, one on each bank of a river, just opposite to each other. One temple is 54 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are  $30^\circ$  and  $60^\circ$  respectively. Find the width of the river and the height of the other temple.



**Sol.** Let AB and CD be the two temples and AC be the river.  
 Then,  $AB = 54$  m.  
 Let  $AC = x$  metres and  $CD = h$  metres.

$$\angle ACB = 60^\circ, \angle EDB = 30^\circ$$

$$AB/AC = \tan 60^\circ = \sqrt{3}$$

$$AC = AB/\sqrt{3} = 54/\sqrt{3} = (54/\sqrt{3} \times \sqrt{3}/\sqrt{3}) = 18\sqrt{3} \text{ m}$$

$$DE = AC = 18\sqrt{3}$$

$$BE/DE = \tan 30^\circ = 1/\sqrt{3}$$

$$BE = (18\sqrt{3} \times 1/\sqrt{3}) = 18 \text{ m}$$

$$CD = AE = AB - BE = (54 - 18) \text{ m} = 36 \text{ m}.$$

$$\text{So, Width of the river} = AC = 18\sqrt{3} \text{ m} = 18 \times 1.73 \text{ m} = 31.14 \text{ m}$$

$$\text{Height of the other temple} = CD = 36 \text{ m}.$$