7. PROBLEMS ON NUMBERS

In this section, questions involving a set of numbers are put in the form of a puzzle. You have to analyze the given conditions, assume the unknown numbers and form equations accordingly, which on solving yield the unknown numbers.

SOLVED EXAMPLES

Ex.1. A number is as much greater than 36 as is less than 86. Find the number.

Sol. Let the number be x. Then, $x - 36 = 86 - x \Rightarrow 2x = 86 + 36 = 122 \Rightarrow x = 61$.
Hence, the required number is 61.

Ex. 2. Find a number such that when 15 is subtracted from 7 times the number, the Result is 10 more than twice the number. (Hotel Management, 2002)

Sol. Let the number be x. Then, $7x - 15 = 2x + 10 \Rightarrow 5x = 25 \Rightarrow x = 5$.
Hence, the required number is 5.

Ex. 3. The sum of a rational number and its reciprocal is 13/6. Find the number. (S.S.C. 2000)

Sol. Let the number be x.
Then, $x + (1/x) = 13/6 \Rightarrow (x^2 + 1)/x = 13/6 \Rightarrow 6x^2 - 13x + 6 = 0$
$\Rightarrow 6x^2 - 9x - 4x + 6 = 0 \Rightarrow (3x - 2)(2x - 3) = 0$
$\Rightarrow x = 2/3$ or $x = 3/2$
Hence the required number is 2/3 or 3/2.

Ex. 4. The sum of two numbers is 184. If one-third of the one exceeds one-seventh of the other by 8, find the smaller number.

Sol. Let the numbers be x and $(184 - x)$. Then,
$(X/3) - ((184 - x)/7) = 8 \Rightarrow 7x - 3(184 - x) = 168 \Rightarrow 10x = 720 \Rightarrow x = 72$.
So, the numbers are 72 and 112. Hence, smaller number = 72.

Ex. 5. The difference of two numbers is 11 and one-fifth of their sum is 9. Find the numbers.

Sol. Let the number be x and y. Then,
$x - y = 11$ ----(i) and $1/5 (x + y) = 9 \Rightarrow x + y = 45$ ----(ii)
Adding (i) and (ii), we get: $2x = 56$ or $x = 28$. Putting $x = 28$ in (i), we get: $y = 17$.
Hence, the numbers are 28 and 17.

Ex. 6. If the sum of two numbers is 42 and their product is 437, then find the absolute difference between the numbers. (S.S.C. 2003)

Sol. Let the numbers be x and y. Then, $x + y = 42$ and $xy = 437$
$x - y = \text{sqrt}[(x + y)^2 - 4xy] = \text{sqrt}[(42)^2 - 4 \times 437] = \text{sqrt}[1764 - 1748] = \text{sqrt}[16] = 4$.
Required difference = 4.

Ex. 7. The sum of two numbers is 16 and the sum of their squares is 113. Find the numbers.

Sol. Let the numbers be x and (15 - *x).*

Then, $x^2 + (15 - x)^2 = 113$ => $x^2 + 225 + X^2 - 30x = 113$

=> $2x^2 - 30x + 112 = 0$ => $x^2 - 15x + 56 = 0$

=> $(x - 7)(x - 8) = 0$ => $x = 7$ or $x = 8$.

So, the numbers are 7 and 8.

Ex. 8. The average of four consecutive even numbers is 27. Find the largest of these numbers.

Sol. Let the four consecutive even numbers be x, x + 2, x + 4 and x + 6.

Then, sum of these numbers = (27 x 4) = 108.

So, $x + (x + 2) + (x + 4) + (x + 6) = 108$ or $4x = 96$ or $x = 24$.

:. Largest number = $(x + 6) = 30$.

Ex. 9. The sum of the squares of three consecutive odd numbers is 2531.Find the numbers.

Sol. Let the numbers be x, x + 2 and x + 4.

Then, $X^2 + (x + 2)^2 + (x + 4)^2 = 2531$ => $3x^2 + 12x - 2511 = 0$

=> $X^2 + 4x - 837 = 0$ => $(x - 27)(x + 31) = 0$ => $x = 27$.

Hence, the required numbers are 27, 29 and 31.

Ex. 10. Of two numbers, 4 times the smaller one is less then 3 times the 1arger one by 5. If the sum of the numbers is larger than 6 times their difference by 6, find the two numbers.

Sol. Let the numbers be x and *y,* such that x > y

Then, $3x - 4y = 5$ ...(i) and $(x + y) - 6(x - y) = 6$ => $-5x + 7y = 6$ …(ii)

Solving (i) and *(ii),* we get: x = 59 and $y = 43$.

Hence, the required numbers are 59 and 43.

Ex. 11. The ratio between a two-digit number and the sum of the digits of that number is 4 : 1.If the digit in the unit's place is 3 more than the digit in the ten's place, what is the number?

Sol. Let the ten's digit be x. Then, unit's digit = $(x + 3)$.

Sum of the digits = $x + (x + 3) = 2x + 3$. Number = $10x + (x + 3) = 11x + 3$.

$11x+3 / 2x + 3 = 4 / 1$ => $11x + 3 = 4(2x + 3)$ => $3x = 9$ => $x = 3$.

Hence, required number = $11x + 3 = 36$.

Ex. 12. A number consists of two digits. The sum of the digits is 9. If 63 is subtracted from the number, its digits are interchanged. Find the number.

Sol. Let the ten's digit be x. Then, unit's digit = (9 - *x).*

Number = $10x + (9 - x) = 9x + 9$.

Number obtained by reversing the digits = $10(9 - x) + x = 90 - 9x$.

therefore, $(9x + 9) - 63 = 90 - 9x$ => $18x = 144$ => $x = 8$.

So, ten's digit = 8 and unit's digit = 1.

Hence, the required number is 81.

Ex. 13. A fraction becomes 2/3 when 1 is added to both, its numerator and denominator.

And ,it becomes 1/2 when 1 is subtracted from both the numerator and denominator. Find the fraction.

Sol. Let the required fraction be x/y. Then,

$x+1 / y+1 = 2 / 3 \Rightarrow 3x - 2y = -1$ …(i) and $x - 1 / y - 1 = 1 / 2$

$\Rightarrow 2x - y = 1$ …(ii)

Solving (i) and (ii), we get : $x = 3$, $y = 5$

therefore, Required fraction= 3 / 5.

Ex. 14. 50 is divided into two parts such that the sum of their reciprocals is 1/ 12. Find the two parts.

Sol. Let the two parts be x and (50 - *x).*

Then, $1 / x + 1 / (50 - x) = 1 / 12 \Rightarrow (50 - x + x) / x (50 - x) = 1 / 12$

$\Rightarrow x^2 - 50x + 600 = 0 \Rightarrow (x - 30) (x - 20) = 0 \Rightarrow x = 30$ or $x = 20$.

So, the parts are 30 and 20.

Ex. 15. If three numbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers)

Sol. Let the numbers be x, *y* and z. Then,

$x + y = 10$ ...*(i)* $y + z = 19$ ...*(ii)* $x + z = 21$ …(iii)

Adding (i) ,*(ii)* and *(iii),* we get: $2 (x + y + z) = 50$ or $(x + y + z) = 25$.

Thus, $x = (25 - 19) = 6$; $y = (25 - 21) = 4$; $z = (25 - 10) = 15$.

Hence, the required numbers are 6, 4 and 15.