9. SURDS AND INDICES

I IMPORTANT FACTS AND FORMULAE I

1. LAWS OF INDICES:

(i) $a^m \times a^n = a^{m+n}$
(ii) $a^m / a^n = a^{m-n}$
(iii) $(a^m)^n = a^{mn}$
(iv) $(ab)^n = a^n b^n$
(v) $(a/b)^n = (a^n / b^n)$
(vi) $a^0 = 1$

2. SURDS: *Let a be a rational number and* n *be a positive integer such that* $a^{1/n} = {}^n\text{sqrt}(a)$ *is irrational. Then* ^{n}sqrt(a) is *called a surd of order* n.

3. LAWS OF SURDS:

(i) ${}^n\sqrt{a} = a^{1/2}$
(ii) ${}^n\sqrt{ab} = {}^n\sqrt{a} * {}^n\sqrt{b}$
(iii) ${}^n\sqrt{a/b} = {}^n\sqrt{a} \ / \ {}^n\sqrt{b}$
(iv) $({}^n\sqrt{a})^n = a$
(v) ${}^m\sqrt{({}^n\sqrt{(a)})} = {}^{mn}\sqrt{(a)}$
(vi) $({}^n\sqrt{a})^m = {}^n\sqrt{a^m}$

I SOLVED EXAMPLES

Ex. 1. Simplify : (i) $(27)^{2/3}$ (ii) $(1024)^{-4/5}$ (iii)$(8 / 125)^{-4/3}$

Sol. (i) $(27)^{2/3} = (3^3)^{2/3} = 3^{(3 * (2/3))} = 3^2 = 9$
(ii) $(1024)^{-4/5} = (4^5)^{-4/5} = 4^{\{5 * ((-4)/5)\}} = 4^{-4} = 1 / 4^4 = 1 / 256$
(iii) $(8 / 125)^{-4/3} = \{(2/5)^3\}^{-4/3} = (2/5)^{\{3 * (-4/3)\}} = (2/5)^{-4} = (5/2)^4 = 5^4 / 2^4 = 625 / 16$

Ex. 2. Evaluate: (i) $(.00032)^{3/5}$ *(ii)l* $(256)^{0.16} \times (16)^{0.18}$.

Sol. (i) $(0.00032)^{3/5} = (32 / 100000)^{3/5}. = (2^5 / 10^5)^{3/5} = \{(2/10)^5\}^{3/5} = (1/5)^{(5 * 3/5)} = (1/5)^3 = 1 / 125$
(ii) $(256)^{0.16} * (16)^{0.18} = \{(16)^2\}^{0.16} * (16)^{0.18} = (16)^{(2 * 0.16)} * (16)^{0.18}$
$= (16)^{0.32} * (16)^{0.18} = (16)^{(0.32+0.18)} = (16)^{0.5} = (16)^{1/2} = 4.$

Ex. 3. *What is the quotient when* $(x^{-1} - 1)$ *is divided by* $(x - 1)$?

Sol. $\dfrac{x^{-1} - 1}{x - 1} = \dfrac{(1/x) - 1}{x - 1} = \dfrac{1 - x}{x} * \dfrac{1}{(x - 1)} = \dfrac{-1}{x}$

Hence, the required quotient is $-1/x$

Ex. 4. If $2^{x-1} + 2^{x+1} = 1280$, *then find the value* of x.

Sol. $2^{x-1} + 2^{X+1} = 1280 \Leftrightarrow 2^{x-1}(1 + 2^2) = 1280$

$$\Leftrightarrow 2^{x-1} = 1280 / 5 = 256 = 2^8 \Leftrightarrow x - 1 = 8 \Leftrightarrow x = 9.$$

Hence, x = 9.

Ex. 5. *Find* the value of $[5(8^{1/3} + 27^{1/3})^3]^{1/4}$

Sol. $[5(8^{1/3} + 27^{1/3})^3]^{1/4} = [5\{(2^3)^{1/3} + (3^3)^{1/3}\}^3]^{1/4} = [5\{(2^{3*1/3})^{1/3} + (3^{3*1/3})^{1/3}\}^3]^{1/4}$

$$= \{5(2+3)^3\}^{1/4} = (5 * 5^3)^{1/4} = 5^{(4*1/4)} = 5^1 = 5.$$

Ex. 6. Find the Value of $\{(16)^{3/2} + (16)^{-3/2}\}$

Sol. $[(16)^{3/2} + (16)^{-3/2} = (4^2)^{3/2} + (4^2)^{-3/2} = 4^{(2*3/2)} + 4^{\{2*(-3/2)\}}$

$$= 4^3 + 4^{-3} = 4^3 + (1/4^3) = (64 + (1/64)) = 4097/64.$$

Ex. 7. *If* $(1/5)^{3y} = 0.008$, *then find the value of* $(0.25)^y$.

Sol. $(1/5)^{3y} = 0.008 = 8/1000 = 1/125 = (1/5)^3 \Leftrightarrow 3y = 3 \Leftrightarrow Y = 1.$

$\therefore (0.25)^y = (0.25)^1 = 0.25.$

Ex. 8. *Find the value* of $\frac{(243)^{n/5} \times 3^{2n+1}}{9^n \times 3^{n-1}}$.

Sol. $\frac{(243)^{n/5} x 3^{2n+l}}{(3^2)^n \times 3^{n-1}} = \frac{3^{(5 * n/5)} \times 3^{2n+l}}{3^{2n} \times 3^{n-1}} = \frac{3^n \times 3^{2n+1}}{32n \times 3^{n-l}}$

$= \frac{3^{n+(2n+1)}}{3^{2n+n-1}} = \frac{3^{(3n+1)}}{3^{(3n-1)}} = 3^{(3n+l)-(3n-l)} = 3^2 = 9.$

Ex. 9. *Find the value Of* $(2^{1/4}-1)(2^{3/4}+2^{1/2}+2^{1/4}+1)$

Sol.

Putting $2^{1/4} = x$, we get :

$(2^{1/4}-1)(2^{3/4}+2^{1/2}+2^{1/4}+1)=(x-1)(x^3+x^2+x+1)$, *where* $x = 2^{1/4}$

$=(x-1)[x^2(x+1)+(x+1)]$

$=(x-1)(x+1)(x^2+1) = (x^2-1)(x^2+1)$

$=(x^4-1) = [(2^{1/4})^4-1] = [2^{(1/4*4)} - 1] = (2-1) = 1.$

Ex. 10. Find the value of $\frac{6^{2/3} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$

Sol. $\frac{6^{2/3} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}} = \frac{6^{2/3} \times (6^7)^{1/3}}{(6^6)^{1/3}} = \frac{6^{2/3} \times 6^{(7 * 1/3)}}{6^{(6 * 1/3)}} = \frac{6^{2/3} \times 6^{(7/3)}}{6^2}$

$=6^{2/3} \times 6^{((7/3)-2)} = 6^{2/3} \times 6^{1/3} = 6^1 = 6.$

Ex. 11. If $x= y^a$, $y=z^b$ *and* $z=x^c$,*then find the value of abc.*

Sol. $z^1 = x^c = (y^a)^c$ [*since* $x= y^a$]

$=y^{(ac)} = (z^b)^{ac}$ [*since* $y=z^b$]

$=z^{b(ac)}= z^{abc}$

$\therefore$ $abc = 1.$

Ex. 12. Simplify $[(x^a / x^b)\wedge(a^2+b^2+ab)] * [(x^b / x^c)\wedge b^2+c^2+bc)] * [(x^c/x^a)\wedge(c^2+a^2+ca)]$

Sol.

Given Expression

$= [\{x^{(o-b)}\}\wedge(a^2 + b^2 + ob)].['(x^{(b-c)}\}\wedge(b^2 + c^2 + bc)].['(x^{(c-a)}\}\wedge(c^2 + a^2 + ca])$

$= [x^{(a-b)(a2+b2+ab)} . x^{(b-c)(b2+c2+bc)} . x^{(c-a)(c2+a2+ca)}]$

$= [x\wedge(a^3-b^3)].[x\wedge(b^3-e^3)].[x\wedge(c^3-a^3)] = x\wedge(a^3-b^3+b^3-c^3+c^3-a^3) = x^0 = 1.$

Ex. 13. *Which is larger* $\sqrt{2}$ or $\sqrt[3]{3}$?

Sol. Given surds are of order 2 and 3. Their L.C.M. is 6. Changing each to a surd of order 6, we get:

$$\sqrt{2} = 2^{1/2} = 2^{((1/2)*(3/2))} = 2^{3/6} = 8^{1/6} = \sqrt[6]{8}$$
$$\sqrt[3]{3} = 3^{1/3} = 3^{((1/3)*(2/2))} = 3^{2/6} = (3^2)^{1/6} = (9)^{1/6} = \sqrt[6]{9}.$$

Clearly, $\sqrt[6]{9} > \sqrt[6]{8}$ and hence $\sqrt[3]{3} > \sqrt{2}$.

Ex. 14. *Find the largest from among* $4\sqrt{6}$, $\sqrt{2}$ *and* $\sqrt[3]{4}$.

Sol. Given surds are of order 4, 2 and 3 respectively. Their L.C,M, is 12, Changing each to a surd of order 12, we get:

$$\sqrt[4]{6} = 6^{1/4} = 6^{((1/4)*(3/3))} = 6^{3/12} = (6^3)^{1/12} = (216)^{1/12.}$$
$$\sqrt{2} = 2^{1/2} = 2^{((1/2)*(6/6))} = 2^{6/12} = (2^6)^{1/12} = (64)^{1/12}.$$
$$\sqrt[3]{4} = 4^{1/3} = 4^{((1/3)*(4/4))} = 4^{4/12} = (4^4)^{1/12} = (256)^{1/12}.$$

Clearly, $(256)^{1/12} > (216)^{1/12} > (64)^{1/12}$

Largest one is $(256)^{1/12}$. i.e. $\sqrt[3]{4}$.